Another Approach

for Non-Relativistic case:

$$L = \frac{1}{2}mv^2 - q(\phi - \vec{v}\cdot\vec{A}) \quad — (A)$$

$$H = \frac{1}{2m}(\vec{P} - q\cdot\vec{A})^2 + q\phi \quad — (B)$$

Defn:

$$L = T - V$$

$$T - KE \qquad V - PE.$$

for Relativistic Particle. $KE = [mc^2 - m_0c^2]$

So; $L = (m - m_0)c^2 - V$

$$L = c^2\left(\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} - m_0\right) - V$$

$$L = -m_0c^2\left(\frac{-1}{\sqrt{1-v^2/c^2}} + 1\right) - V$$

$$= -m_0c^2\left(1 - \frac{1}{\sqrt{1-v^2/c^2}}\right) - V$$

$$= -m_0c^2\left[1 - (1-v^2/c^2)^{-1/2}\right] - V \quad \text{(using Binomial expansion)}$$

$$\boxed{L \approx -m_0c^2\left(\sqrt{1-\frac{v^2}{c^2}}\right) - V} \quad — (1)$$

$$V = q\phi - q(\vec{v}\cdot\vec{A})$$

$$\boxed{L = -m_0c^2\sqrt{1-v^2/c^2} + q\phi - q(\vec{v}\cdot\vec{A})}$$

The generalized momentum is given by

$$P_\alpha = \frac{\partial L}{\partial \dot{q}_\alpha} = \frac{\partial L}{\partial \dot{x}_\alpha} = \frac{\partial L}{\partial v_\alpha} \quad k^{th} \text{ component of generalized mo}$$

$$p = \frac{\partial}{\partial v}\left[-m_0c^2\sqrt{1-\frac{v^2}{c^2}} - q\phi + q(vA)\right]$$

$$= -m_0c^2 \frac{1}{2}\left(1-\frac{v^2}{c^2}\right)^{-1/2}\cdot\left(-\frac{2v}{c^2}\right) - 0 + qA$$

$$= \frac{m_0 v}{\sqrt{1-v^2/c^2}} + qA$$

$$\boxed{p = mv + qA} \quad \text{—} \quad (2)$$

$$\boxed{\vec{p} = m\vec{v} + q\vec{A}} \quad \text{—} \quad (2)$$

Defn of Hamiltonian :-

$$H = \sum_\alpha p_\alpha \dot{q}_\alpha - L$$

$$= \vec{P}\cdot\vec{v} - L$$

$$= (m\vec{v} + q\vec{A})\cdot\vec{v} - L$$

$$= (m\vec{v} + q\vec{A})\cdot\vec{v} + m_0c^2\sqrt{1-\frac{v^2}{c^2}} + q\phi - q(\vec{v}\cdot\vec{A})$$

$$= m\vec{v}\cdot\vec{v} + q(\vec{v}\cdot\vec{A}) + m_0c^2\sqrt{1-v^2/c^2} + q\phi - q(\vec{v}\cdot\vec{A})$$

$$= mv^2 + m_0c^2\sqrt{1-v^2/c^2} + q\phi$$

$$= \frac{m_0}{\sqrt{1-v^2/c^2}}v^2 + m_0c^2\sqrt{1-v^2/c^2} + q\phi$$

$$= \frac{m_0v^2 + m_0c^2 - m_0v^2}{\sqrt{1-v^2/c^2}} + q\phi$$

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$$\underbrace{\frac{m_0 c^2}{\sqrt{1-v^2/c^2}}}_{\text{Effective K.E.}}$$

$$H = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}} + q\phi$$

$$H = mc^2 + q\phi$$

$$\boxed{H = T^* + q\phi} \quad \text{--- (3)}$$

$$T^{*2} = \frac{m_0^2 c^4}{1-v^2/c^2} = \frac{m_0^2 c^2}{1-v^2/c^2}\left(v^2 + c^2 - v^2\right)$$

$$= \frac{m_0^2 c^2}{1-v^2/c^2}\left[v^2 + c^2(1-v^2/c^2)\right]$$

$$= \frac{m_0^2 c^2 \, v^2}{(1-v^2/c^2)} + m_0^2 c^4$$

$$= \frac{m_0^2 c^2 v^2}{(1-v^2/c^2)} + m_0^2 c^4$$

$$\boxed{T^{*2} = m^2 c^2 v^2 + m_0^2 c^4} \quad \text{--- (4)}$$

from (2) $\quad mv = p - qA$

$$T^{*2} = (p-qA)^2 c^2 + m_0^2 c^4$$

$$\boxed{T^* = \sqrt{(p-qA)^2 c^2 + m_0^2 c^4}} \quad \text{--- (5)}$$

$$\boxed{H = \sqrt{(p-qA)^2 c^2 + m_0^2 c^4} + q\phi.} \quad \text{--- (6)}$$

Relativistic Lagrangian for free Relativistic particle

free Particle $(V=0)$

$$S = \int_0^T L\gamma\, d\tau$$

Action Function $S = \int_0^T L\gamma\, dt$ — τ proper time.

→ must remain invariant under Lorentz transformation. (when we move from one frame to another)

$dt = \gamma\, d\tau$ — τ proper time

$\gamma = \sqrt{1 - v^2/c^2}$

$L\gamma = \text{const}$. const : — mass (m) × c^2 — key

$L = \frac{\text{const}}{\gamma}$ — energy

γ — dimensionless

const — must have unit of energy

So; $\boxed{L = \frac{\alpha m c^2}{\gamma}}$ α — dimensionless const.

only way to built const. energy $= mc^2$

$$p = \frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial v}$$

$$L = \alpha \sqrt{1 - \frac{v^2}{c^2}}\, mc^2 = -\alpha m \gamma \dot{x}$$

for NR: $v << c \Rightarrow \gamma \simeq 1 \Rightarrow p = m\dot{x}$ for ~~[illegible]~~

So; $\boxed{L = \frac{-mc^2}{\gamma}}$

$$p = \alpha m c^2 \frac{-2\frac{v}{c^2}}{2\sqrt{1 - v^2/c^2}} = -\alpha m \gamma v$$

in NR limit: $v << c \Rightarrow \gamma \approx 1$

$\boxed{p = m\dot{x}}$ if $\alpha = -1$

So; $\boxed{L = -\frac{mc^2}{\gamma}}$

The translational invariance of space rules out position. Only invariant we have is $p^{\mu}p_{\mu}$ so. $\gamma L \propto p^{\mu}p_{\mu} = m^2c^2 = \gamma L = C$.

Lagrangian for a relativistic free particle -

As per defn $L = T - V$

Question - is it true for relativistic case.

(1) ~~Newt~~ → equ of motion (Newtonian) is capable ~~of~~ explaining motion of charged particle — (But) (without radiation)

(2) → Lagrangian → useful to understand this part. (free formulation of N.M)

(3) Newtonian Mech — force — vector.
Lagrangian — scalar function treatment is easier.

~~at the same time~~ ~~Newtonian Mechanics~~ it ~~would be~~ difficult to switch from one ~~frame of reference~~ coordinate system to another polar and spherical coordinates.

Lagrangian offers broad basis → Extended to other physical Mechanics.
Q.M, QED etc..

\# → Lagrangian treatment is based on the principle of Least action or Hamilton's principle.

→ System is described by a set of 'n' generalized coordinates $[q_i(t)]$ and velocities $[\dot{q}_i(t)]$
(n) — degree of freedom.

→ Lag. is a scalar function of $q_i(t)$ and $\dot{q}_i(t)$ and time as well.

→ The action 'A' is defined as the time integral of

Lagrangian 'L' along a possible path of the system :

→ Principle of Least action states —
Motion of system is such that in going from a config. 'a' at time t_1 to configuration $\underline{b}$ at time t_2 the action —

$$A = \int_{t_1}^{t_2} L(q_i(t), \dot{q}_i(t)) \, dt$$ is an extremum

→ By considering small variation of the coordinates and velocities away from the actual path and requiring $\boxed{\delta A = 0}$ we obtain Euler-Lagrangian equ of motion.

→ $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \quad i = 1, 2, \ldots n$$

q_i and $\dot{q}_i$ are generalized (coordinates) positions and velocities.

Then the generalized momenta are defined by $\boxed{p_i = \frac{\partial L}{\partial \dot{q}_i}}$

and Hamiltonian by : (generalized coordinates and momentum)

$$\boxed{H(q_i, p_i, t) = \sum_{i=1}^{n} p_i \dot{q}_i - L(q_i, \dot{q}_i, t)}$$

Hamiltonian equ of motions — which are completely equivalent to the Euler Lagrange Equ →

can be derived from them –

$$\frac{\partial H}{\partial q_i} = -\dot{p}_i \quad ; \quad \frac{\partial H}{\partial p_i} = \dot{q}_i$$

** Applying this to some simple NR. case of a free particle ($V=0$)

$L = T - V$

$$L(x,\dot{x},t) = T = \frac{1}{2}m\dot{x}^2.$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \qquad \frac{\partial L}{\partial x} = 0 \Rightarrow$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \quad \text{is } \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{d}{dt}m\dot{x} = 0$$

or; $\boxed{m\ddot{x} = 0}$

Also; $\frac{\partial L}{\partial \dot{x}} = p$; so; $\boxed{p = m\dot{x}}$

Hence; $H = p\dot{x} - L$

$$= m\dot{x}\dot{x} - \frac{1}{2}m\dot{x}^2$$

or; $H = \frac{1}{2}m\dot{x}^2 = \frac{(m\dot{x})^2}{2m} = \frac{p^2}{2m} = T$

Lagrangian & Hamiltonian of a particle in a potential $V(x)$

$$L(x,\dot{x},t) = T - V(x) = \frac{1}{2}m\dot{x}^2 - V(x)$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \qquad \frac{\partial L}{\partial x} = -\frac{\partial V}{\partial x}$$

So; from Eulers eqn → $m\ddot{x} = -\frac{\partial V}{\partial x} = F(x)$

$$\boxed{F(x) = m\ddot{x}}$$

then; $p = \frac{\partial L}{\partial \dot{x}} = m\dot{x}$ momentum.

Lagrangian for a charged Particle in EMF.

for a charged particle in an Ext. EMF, the eq of motion and rate of change of energy are –

$$\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\frac{d\vec{W}}{dt} = q\vec{v}\cdot\vec{E}$$

{ $\vec{W}$ – Energy
$\vec{E}$ – electric field intensity }

[since – Mag. force ⊥r to velocity No work is done]

Together;

$$\frac{dU^\alpha}{d\tau} = \frac{q}{m} F^{\alpha\beta} U_\beta$$

$U^\alpha = (\gamma c, \gamma v)$ – four velocity

$F^{\alpha\beta}$ – force field tensor.

General requirement that $[\gamma L]$ be invariant allow us determine 'L' in emf.

$$L_{Total} = L_{free} + \underbrace{L_{int.}}$$

(interaction Lag. contain info about force & fields)

for invariance –

$$\delta A_{int} = \delta \int_{t_1}^{t_2} L_{int}\, dt = 0$$

In NR. $\boxed{L = T - V}$.

$v << c$. ; $V \approx q\phi$

[Slow moving charge particle are influenced by $\vec{E}$ only] ϕ – scalar potential.

in this limit –

$$\gamma L_{int} = -\gamma q\phi = -q\phi\, W/mc^2$$

$$= -\frac{q}{m} P_0 A_0$$

$F = q[E + v \times B]$

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~~Conservative forces~~

Lagrangian for a relativistic charged particle in ext. Mag. fields.

Eqns of motion — $\frac{dp}{dt} = e\left[E + \frac{u}{c} \times B\right]$

$$\frac{dE}{dt} = e u \cdot E \qquad \text{eq-1}$$

for a charged particle 'e' in an ext. field E & B can be written in the covariant form.

$$\frac{dU^{\alpha}}{d\tau} = \frac{e}{mc} F^{\alpha\beta} U_{\beta}$$

m — mass

τ - proper time

$U^{\alpha} = (\gamma c, \gamma u) = \frac{p^{\alpha}}{m}$ } 4 velocity of the particle.

Eq (1) are sufficient to describe the general

Lagrangian $[L = T - U]$

U — velocity dependent potential.

force on charged in field — $\vec{F}$ Lorentz force (due to both $\vec{E}$ & $\vec{B}$)

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$
$$q[\vec{E} + (\vec{v} \times B)]$$

But $\vec{B}$ = curl of potential $= \vec{\nabla} \times \vec{A}$

$\vec{A}$ — Mag. vector potential

$$L = \alpha mc^2 \left(1 - \frac{v^2}{c^2}\right)^{1/2}$$

$$= \alpha m \cancel{c^2} \frac{1}{\cancel{2}} \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \cdot -\frac{\cancel{2}v}{\cancel{c^2}}$$

$$- \underline{\alpha m v \gamma} \cdot = \underline{\alpha m \gamma \dot{x}}$$

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$$\vec{\nabla} \times \vec{E} + \frac{\partial B}{\partial t} = 0$$

so; $$\vec{\nabla} \times \vec{E} + \frac{\partial (\vec{\nabla} \times \vec{A})}{\partial t} = 0$$

$$\vec{\nabla} \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0 \qquad -(4)$$

from vector calculus we know that curl of gradient of some scalar function ϕ is always zero.

$$\vec{\nabla} \times (\nabla \phi) = 0$$

or; $$\vec{\nabla} \times (-\nabla \phi) = 0 \qquad -(5)$$

comparing (4) & (5)

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla \phi$$

or; $$\vec{E} = -\nabla \phi - \frac{\partial \vec{A}}{\partial t} \qquad -(6)$$

so; $$\vec{F} = q\left[-\nabla \phi - \frac{\partial A}{\partial t} + (\vec{v} \times \vec{\nabla} \times \vec{A})\right]$$

Now

Taking x components of F

$$F_x = q\left[-\frac{\partial \phi}{\partial x} - \frac{d}{dt} A_x + (\vec{v} \times \vec{\nabla} \times \vec{A})_x\right]$$

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

also. $$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

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Let us find 'x' component of $[\vec{v}\times(\vec{\nabla}\times\vec{A})]_x$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{j}\left(-\frac{\partial}{\partial x}A_z + \frac{\partial}{\partial z}A_x\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\vec{v}\times(\vec{\nabla}\times\vec{A}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z} & \frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x} & \frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y} \end{vmatrix} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ \frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z} & \frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x} & \frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y} \end{vmatrix}$$

$$[\vec{v}\times\vec{\nabla}\times\vec{A}]_x = v_y\left(\frac{\partial A_y}{\partial x} - \frac{\partial}{\partial y}A_x\right) - v_z\left(-\frac{\partial A_z}{\partial x} + \frac{\partial A_x}{\partial z}\right)$$

$$= v_y\frac{\partial A_y}{\partial x} - v_y\frac{\partial A_x}{\partial y} + v_z\frac{\partial A_z}{\partial x} - v_z\frac{\partial A_x}{\partial z}.$$

Adding & substracting $v_x\frac{\partial A_x}{\partial x}$;

$$v_y\frac{\partial}{\partial x}A_y + v_z\frac{\partial A_z}{\partial x} - v_y\frac{\partial}{\partial y}A_x - v_z\frac{\partial A_x}{\partial z} + v_x\frac{\partial A_x}{\partial x} - v_x\frac{\partial A_x}{\partial x}.$$

$$\frac{\partial}{\partial x}(v_x A_x + v_y A_y + v_z A_z) -$$

$$\frac{\partial}{\partial x}(\vec{v}\cdot\vec{A})$$

Again. $A_x = A_x(x, y, z, t)$

$$-\frac{d}{dv_x}(\vec{v}\cdot\vec{A}) = -A_x \qquad F_x = q\left[-\frac{\partial}{\partial x}(\phi - \vec{v}\cdot\vec{A}) + \frac{d}{dt}\right.$$

$$\frac{d}{dv_x}(\phi - \vec{v}\cdot\vec{A}) = -A_x \qquad \boxed{F_x = -\frac{\partial U}{\partial x} + \frac{\partial}{\partial t}\frac{\partial U}{\partial v_x}}$$

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$$\text{or } -\frac{dA_x}{dt} = \frac{d}{dt}\left[\frac{d}{dv_x}(\phi - \vec{v}\cdot\vec{A})\right]$$

$$U = q(\phi - \vec{v}\cdot\vec{A})$$

velo

e.p.A

List of Supervisor

$$\boxed{L = T - U}$$

$$L = T - q(\phi - u\cdot\vec{A})$$

$$L = \tfrac{1}{2}mv^2 - 300 \text{ (St. Reg.)}$$

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$$\boxed{P_x = mv - qA_x}$$

$$L = T - U = \tfrac{1}{2}mv^2 - q\phi + \dots$$

$$P_x = \frac{\partial L}{\partial v_x} = mv - q\frac{\partial}{\partial v_x}\dots$$

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1.1 —
2.1 — Saju
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4.1
4.2

Maths - (7).

$$A_x = A_x(x, y, z, t)$$

$$\frac{dA_x}{dt} = \frac{\partial A_x}{\partial x}\frac{dx}{dt} + \frac{\partial A_x}{\partial y}\frac{dy}{dt} + \frac{\partial A_x}{\partial z}\frac{dz}{dt} + \frac{\partial A_x}{\partial t}$$

$$\frac{dA_x}{dt} - \frac{\partial A_x}{\partial t} = \frac{\partial A_x}{\partial x}v_x + \frac{\partial A_x}{\partial y}v_y + \frac{\partial A_x}{\partial z}v_z + \frac{\partial A_x}{\partial t}$$

$$\vec{v}\times(\vec{\nabla}\times\vec{A}) = \frac{\partial}{\partial x}(\vec{v}\cdot\vec{A}) - \left(\frac{dA}{dt} - \frac{\partial A}{\partial t}\right)$$

$$F_x = q\left(-\frac{\partial \phi}{\partial x} - \frac{\partial A_x}{\partial t} + \frac{\partial}{\partial x}(\vec{v}\cdot\vec{A}) - \frac{dA}{dt} + \frac{\partial A}{\partial t}\right)$$

$$= q\left[-\frac{\partial}{\partial x}(\phi - \vec{v}\cdot\vec{A}) - \frac{dA_x}{dt}\right]$$

$$\frac{d}{dv_x}(\vec{v}\cdot\vec{A}) = \frac{d}{dv_x}(v_xA_x + v_yA_y + v_zA_z) = A_x$$

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The total Lagrangian for a charged particle -

$$L = L_{free} + L_{int}$$

$$L = -mc^2\sqrt{1-\frac{v^2}{c^2}} - q\phi + q\vec{v}\cdot\vec{A} \quad \text{—} (1)$$

$\boxed{\text{Equ of Motion}}$ using Euler-Lagrange equ

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q} = 0$$

and.

$$\frac{dA_i(x,t)}{dt} = \frac{\partial A_i}{\partial t} + \frac{\partial A_i}{\partial x}\frac{dx_j}{dt} \quad \text{—} (2)$$

$$= \frac{\partial A_i}{\partial t} + (u\cdot\nabla)A_i$$

~~We obtain~~ -

$$\frac{d\vec{p}}{dt} = \qquad \text{calculation}$$

from eq (1)

$$\frac{\partial L}{\partial x_i} = -q\frac{\partial \phi}{\partial x_i} + q\frac{\partial}{\partial x_i}(\vec{v}\cdot\vec{A})$$

$$\frac{\partial L}{\partial v_i} = qA_i - (mc^2)\left(-\frac{v_i}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

$$= \cancel{qA_i - (mc^2)\gamma}$$

$$= qA_i + mv_i\gamma$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v_i}\right) = q\frac{dA_i}{dt} + \frac{dP_i}{dt}$$

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Now A_i is the vector potential of the particle and depends on its position $\vec{x}$, which itself is a function of time t so:

$$\frac{dA_i(x,t)}{dt} = \frac{\partial A_i}{\partial t} + \frac{\partial A_i}{\partial x_j}\frac{dx_j}{dt}$$

$$= \frac{\partial A_i}{\partial t} + (\vec{v}\cdot\vec{\nabla})A_i$$

Substituting all these in eq (2) we have,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v_i}\right) - \frac{\partial L}{\partial x_i} = q\left[\frac{\partial A_i}{\partial t} + (\vec{v}\cdot\vec{\nabla})A_i\right]$$

$$+ \frac{dp_i}{dt} + q\frac{\partial \phi}{\partial x_i} - q\frac{\partial}{\partial x_i}($$

$$\frac{d\vec{p}_i}{dt} = -q\left[\frac{\partial A_i}{\partial t} + (\vec{v}\cdot\vec{\nabla})A_i\right] - q\frac{\partial \phi}{\partial x_i} + q\frac{\partial}{\partial x_i}(\vec{v}\cdot A$$

$$\frac{d\vec{p}}{dt} = \underbrace{-q\left[\frac{\partial \vec{A}}{\partial t} + \nabla\phi\right]}_{\text{Electric field}} + \underbrace{q\left[\vec{\nabla}(\vec{v}\cdot\vec{A}) - (\vec{v}\cdot\right.}_{\vec{u}\times B.}$$

i.e. $\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi$

$$\vec{v} \times \vec{B} = \vec{v} \times (\vec{\nabla} \times \vec{A}) = \underline{\vec{\nabla}(\vec{v}\cdot\vec{A}) - (\vec{v}\cdot\vec{\nabla})\vec{A}}$$

Thus, again we get:

$$\boxed{\frac{dp}{dt} = q(\vec{E} + \vec{v} \times \vec{B})}$$

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Manifestly covariant treatment -

* For manifestly covariant treatment vector $(\vec{x}, \vec{u})$ — must be replaced by four vector form (x^α, u^α)

$$x^\alpha = ct, x \quad , \quad u^\alpha = \gamma c \text{ or } \gamma u$$

Then free particle Lagrangian -

$$L_{free} = -\frac{mc}{\gamma}\sqrt{U^\alpha U_\alpha}$$

Action integral -

$$A = -mc\int_{\tau_1}^{\tau_2} \sqrt{U^\alpha U_\alpha}\, d\tau$$

→ Of the four components of the 4 velocity only 3 are really independent

→ Because of the constraint equ -

$$U^\alpha U_\alpha = c^2 \Rightarrow U^\alpha \frac{dU_\alpha}{d\tau} = 0$$

This has to be incorporated into the equ of motion we cannot freely vary this action to find equ motion - the integral -

$$\sqrt{U^\alpha U_\alpha}\, d\tau = \sqrt{g^{\alpha\beta} U_\alpha U_\beta}\, d\tau = \sqrt{g^{\alpha\beta}\frac{dx_\alpha}{d\tau}\frac{dx_\beta}{d\tau}}$$

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The force acting

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$$\frac{dA_x}{dt} = \frac{\partial A_x}{\partial x}\frac{dx}{dt} + \frac{\partial A_y}{\partial y}\frac{dy}{dt} + \frac{\partial A_z}{\partial z}\frac{dz}{dt} + \frac{\partial A_x}{\partial t}\cancel{\frac{dt}{dt}}$$

$$\frac{dA_x}{dt} - \frac{\partial A_x}{\partial t} = \frac{\partial A_x}{\partial x}v_x + \frac{\partial A_y}{\partial y}v_y + \frac{\partial A_z}{\partial z}v_z$$

So; $$\left[\vec{v} \times (\vec{\nabla}\times\vec{A})\right]_x = \frac{\partial}{\partial x}(\vec{v}\cdot\vec{A}) - \left(\frac{d\vec{A}}{dt} - \frac{\partial\vec{A}}{\partial t}\right)$$

So;

$$F_x = q\left[-\frac{\partial\phi}{\partial x} - \cancel{\frac{\partial A_x}{\partial t}} + \frac{\partial}{\partial x}(\vec{v}\cdot\vec{A}) - \frac{dA_x}{dt} + \cancel{\frac{\partial A_x}{\partial t}}\right]$$

$$= q\left[-\frac{\partial}{\partial x}(\phi - \vec{v}\cdot\vec{A}) - \frac{dA_x}{dt}\right] \quad - (12)$$

Now; to represent A in terms of v & A

$$\frac{d}{dv_x}(\vec{v}\cdot\vec{A}) = \frac{d}{dv_x}(v_xA_x + v_yA_y + A_zv_z)$$

$$= A_x.$$

So, $$-\frac{d}{dv_x}(\vec{v}\cdot\vec{A}) = -A_x$$

since ϕ is scalar, independent of ϕ

$$+\frac{d}{dv_x}(\phi - \vec{v}\cdot\vec{A}) = -A_x.$$

So; $$-\frac{d}{dt}A_x = \frac{d}{dt}\left[\frac{d}{dv_x}(\phi - \vec{v}\cdot\vec{A})\right]$$

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So,

$$F_x = q\left[-\frac{\partial}{\partial x}(\phi - \vec{V}\cdot\vec{A}) + \frac{d}{dt}\left\{\frac{\partial}{\partial v_x}(\vec{v}\cdot\right.\right.$$

Comparing with. $f_x = -\frac{\partial U}{\partial x} + \frac{\partial}{\partial t}\frac{\partial U}{\partial v_x}$.

Generalized potential –

$$U = q(\phi - \vec{v}\cdot\vec{A})$$ – velocity dependent generalised pot

$L = T - U$.

$$\boxed{L = T - q(\phi - \vec{v}\cdot\vec{A})} = \frac{1}{2}mv^2 + q$$

Momentum: $p_x = \frac{\partial L}{\partial \dot{x}} = mv_x + q A_x$

Similarly $p = \underline{mv} + \cancel{q(\vec{v}\cdot\vec{A})}\ q\vec{A}$

L. moment. + qA.

Hamiltonian:

$$H = \sum_i p_\alpha \dot{x}_\alpha - L = \sum_i \underline{p_x v_x} - L$$

$$H = \frac{1}{2m}(p - qA)^2 + q\phi.$$

$$= \vec{P}\cdot\vec{v} - L$$

$$= (mv + q\vec{A})\cdot\vec{v} - \left[\frac{1}{2}mv^2 - q\phi\right.$$

$$mv^2 + q(\vec{v}\cdot\vec{A}) - \frac{1}{2}mv^2 + q\phi$$

$$\frac{1}{2}mv^2 + q\phi.$$

$$= \frac{1}{2}\frac{(mv)^2}{m} + q\phi = \frac{1}{2}\frac{(p - qA)^2}{m}$$

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$$L = \frac{1}{2} m u^2 - V$$

$$= \frac{1}{2} m \dot{x}^2 - V$$

$$P_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x} = \frac{m_0 \dot{x}}{\sqrt{1-\dot{x}^2/c^2}}$$

$$P_x = \frac{m_0 \dot{x}}{\sqrt{1-v^2/c^2}} = \frac{\partial L}{\partial \dot{x}}$$

$$\int_0^L \partial L = \int \frac{m_0 \dot{x}}{\sqrt{1-\frac{\dot{x}^2+\dot{y}^2+\dot{z}^2}{c^2}}} \partial \dot{x}$$

$$1 - \frac{\dot{x}^2+\dot{y}^2+\dot{z}^2}{c^2} = a$$

$$-\frac{2\dot{x}d\dot{x}}{c^2} = da$$

$$L = \int - m_0 \cancel{\dot{x}} \frac{da\, c^2}{2\cancel{\dot{x}}} \cdot \frac{1}{\sqrt{a}}$$

$$= \int - \frac{m_0 c^2}{2} \frac{da}{\sqrt{a}}$$

$x^{-1/2} dx$

$\frac{x^{-1/2+1}}{1}$

$$-\frac{m_0 c^2}{\cancel{2}} \cancel{2} \sqrt{a} = -m_0 c^2 \sqrt{a}$$

$$m_0 c^2 \sqrt{1-\beta^2} \quad \gamma$$

$$L = -m_0 c^2 \sqrt{1-\beta^2} - V$$

$H = \frac{m_0 c^2}{\sqrt{1-\beta^2}} +$

$H = mc^2 +$

$H = E +$

$$H = \sum_i p_i \dot{q}_i - L$$

$$= \frac{m_0 \dot{x}^2}{\sqrt{1-\beta^2}} + 0 + 0 + m_0 c^2 \sqrt{1-\beta^2} + V$$

$$= \frac{m_0 v^2}{\sqrt{1-\beta^2}} + \frac{m_0 c^2 (1 - u^2/c^2)}{\sqrt{1-v^2/c^2}} + V$$

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$$H = \frac{M_0 c^2}{\sqrt{1-u^2/c^2}} + V$$

$H = E + V.$

$$\boxed{H = \sqrt{P^2c^2 + M_0^2 c^4}}$$

for R.. term.

$M \rightarrow m_0 c^2$

$$L = \frac{1}{2} m u^2 + U$$

$E = - M_0$