STATISTICAL THERMODYNAMICS
Part - B
Lecture – 3: Quantum Statistics
(M.Sc. Sem –II/ Chemistry, Paper -III, Unit – II)

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Statistical Thermodynamics

Contents:

- **Recapitulation: Maxwell – Boltzmann Distribution Law**
- **Limitations of Maxwell – Boltzmann Statistics**
- **Bose – Einstein Statistics**
- **Fermi – Dirac Statistics**
- **Comparison of the three Statistics**

Treatment

STATISTICAL THERMODYNAMICS

Bulk / Macroscopic System

Nature of microscopic constituents & Occupancy of the energy level

Different Statistics

PARTICLE STATISTICS

CLASSICAL STATISTICS

Internal structure ignored

Particles distinguishable

MAXWELL – BOLTZMANN STATISTICS

QUANTUM STATISTICS

Internal structure considered

Particles indistinguishable

BOSE – EINSTEIN STATISTICS

FERMI –DIRAC STATISTICS

No restriction on occupancy

Restriction on occupancy

Integral Spin

Half Integral Spin

MAXWELL – BOLTZMANN DISTRIBUTION : Classical Statistics

- Bulk/ macroscopic system with the following conditions:
  - Consist of **N distinguishable particles** with **total energy E** at **temperature T**
  - **No interaction** between particles
  - **No restriction** on **Occupancy** of energy levels
  - Total number of particles **N** and energy **E must remain constant**

    $\Sigma N_i = N$ **and** $\Sigma N_i \varepsilon_i = E$; $\Sigma \delta N_i = 0$ **and** $\Sigma \varepsilon_i \delta N_i = 0$
  - Such particles are called **Boltzmannons** or **Maxwellons** e.g. System composed of gas
  - No. of ways of achieving :

$$\boxed{W = \frac{N!}{N_0! N_1! N_2! \ldots\ldots}}$$

  - Maxwell- Boltzmann Distribution Law:

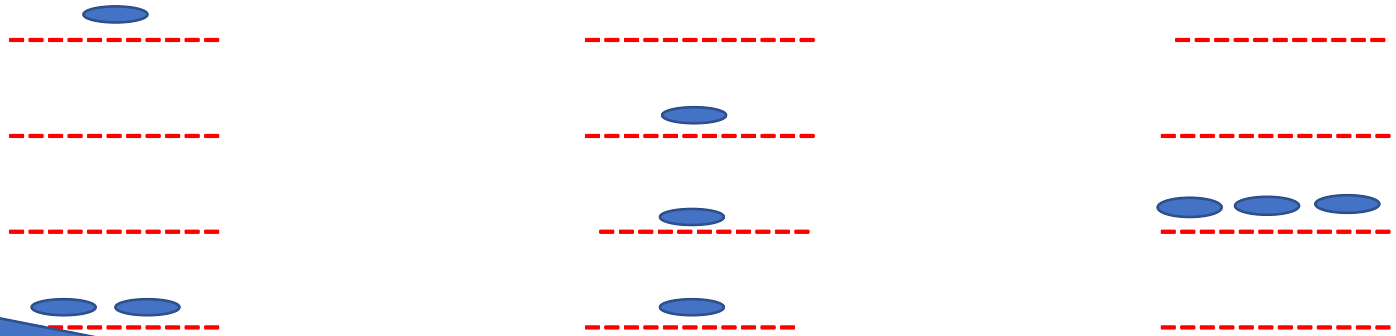
$$\boxed{N_i = g_i e^{-\alpha} e^{-\varepsilon_i/kT}}$$

Limitations of Maxwell – Boltzmann Distribution law

- Maxwell- Boltzmann statistics is based on Classical Mechanics: **Classical Statistics**, so is valid only within classical limit
- **Not valid** at very low temperature and very high particle density, where quantum effects become significant
- Satisfactorily explains **pressure, temperature**, etc. of gaseous systems
- But can we distinguish between gas molecules? In the light of Quantum theory, this leads to that this law is only an approximation & is valid for **gases at comparatively low density**
- Couldn't explain some experimental results like black body radiation distribution, specific heat at low temperature, etc.
- In case of **photon gas**, according to M-B distribution there is continuous no. of photons per unit range of frequency as it increases which is contradicted by Planck's law
- **The molar heat capacity of a metal** is 3R but according to M-B Statistics free electron contributes 3R/2

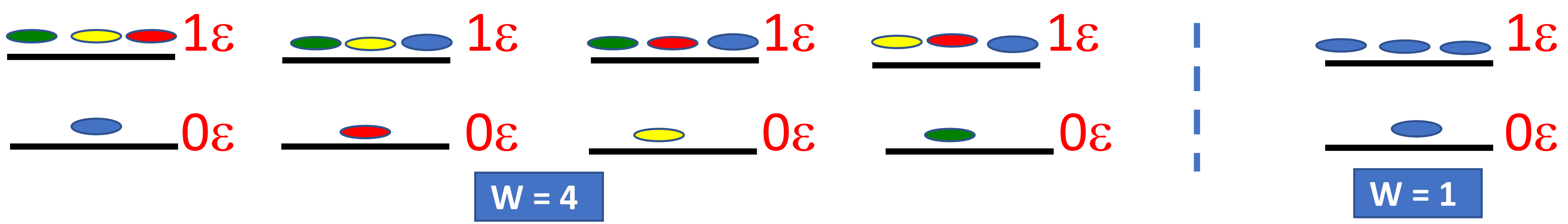
BOSE - EINSTEIN STATISTICS : DERIVATION

- Suppose we have a bulk/ macroscopic system with the following conditions:
  - Consist of **N indistinguishable particles** with **total energy E** at **temperature T**
  - Total number of particles and energy must remain conserved
  - **No restriction** on **Occupancy** of energy states
  - **Internal structure** taken into consideration
  - Particle have zero or integral spin eg. Photon, mesons, ^{4}He, ^{2}H, ………
  - The wave function is symmetric
  - Such particles are called **Bosons**

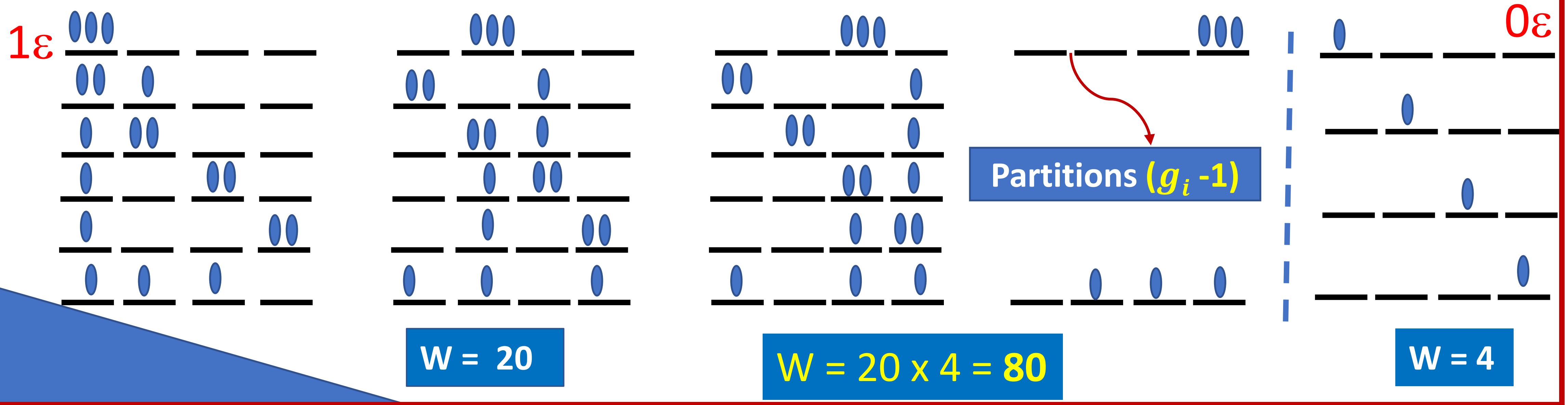


- Consider four particles distributed in two energy levels in such a way that there are
- **3** particles in 1ε and **1** particle in 0ε then no. of microstates according to

Maxwell- Boltzmann Statistics | Bose –Einstein Statistics



- Now if each energy levels have g_i = **4** energy states, then no. of microstates will be



Derivation of Bose – Einstein Distribution

- Most Probable Distribution **???**
  - The one with maximum no. of microstates or gives Maximum Thermodynamic Probability
  - **W_{max} : $\delta W(N) = 0$ or $\delta \ln W(N) = 0$**
- Consider the distribution of **N** identical and indistinguishable particles among various energy levels - **$\varepsilon_0, \varepsilon_1, \varepsilon_2$ ....** having **g_0, g_1, g_2, ...** energy **states** with Total energy **E** at temperature **T**
- Total number of particles N and Total energy E remains **constant**

$$\Sigma N_i = N \quad \textbf{and} \quad \Sigma N_i \varepsilon_i = E$$

$$\Sigma \delta N_i = 0 \quad \textbf{and} \quad \Sigma \varepsilon_i \delta N_i = 0$$

- **N_0** particles are present in ε_0 energy level with g_0 energy states, **N_1** in ε_1 energy level with g_1 energy states, **N_2** in ε_2 energy level with g_2 energy states, **N_i** in ε_i energy level with g_i energy states, ... with **no restriction**

Derivation of Bose – Einstein Distribution

- Suppose there are N_i particles are present in ε_i energy level
- The energy level ε_i is considered to be degenerate, in which there are g_i energy states
- $(g_i - 1)$ partitions are required to place the N_i particles in g_i energy states
- **No restriction** on no. of particles occupying each energy state
- Permutations of N_i particles and $(g_i - 1)$ partitions simultaneously is given by

$$(N_i + g_i - 1)!$$

- Particles are **identical and indistinguishable**
- Permutations of N_i particles amongst themselves and $(g_i - 1)$ partitions amongst themselves has to be included
- Actual no. in which N_i particles may be allocated in g_i states will be given by

$$\frac{(N_i + g_i - 1)!}{N_i!\,(g_i - 1)!}$$

Example

The Thermodynamic Probability will be given by:

$$W = \prod \frac{(N_i + gi - 1)!}{N_i!(g_i - 1)!}$$

- $\mathbf{W} = \frac{(N_i + gi - 1)!}{N_i!(g_i - 1)!} = \frac{(3+4-1)!}{3!(4-1)!} = \frac{6\,!}{3!(3)!} = \frac{\cancel{6} \times 5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1}(\cancel{3} \times \cancel{2} \times \cancel{1})} = \mathbf{20}$

- $\mathbf{W} = \frac{(N_i + gi - 1)!}{N_i!(g_i - 1)!} = \frac{(1+4-1)!}{1!(4-1)!} = \frac{4\,!}{1!(3)!} = \frac{4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{1(\cancel{3} \times \cancel{2} \times \cancel{1})} = \mathbf{4}$

- $\mathbf{W} = \prod \frac{(N_i + gi - 1)!}{N_i!(g_i - 1)!} = \mathbf{20 \times 4 = 80}$

Derivation of Bose – Einstein Statistics

- The Thermodynamic Probability is given by:

$$W = \prod \frac{(N_i+g_i-1)!}{N_i!(g_i-1)!} \qquad \text{.......................... (1)}$$

- Taking logarithm, we get

$$ln\ W = \Sigma\{ln(N_i + g_i - 1)! - [ln(N_i!) + ln\ (g_i - 1)!]\} \qquad \text{.......................... (2)}$$

- Neglecting Unity as compared to g_i

$$ln\ W = \Sigma\ \{ln(N_i + gi)! - [ln(N_i!) + ln\ (g_i)!]\} \qquad \text{.......................... (3)}$$

- Applying **STIRLING'S Approximation:** $ln(N_i!) = N_i lnN_i - N_i$, we get

$$ln\ W = \Sigma\{[(N_i + g_i)\ ln(g_i + N_i) - (N_i + g_i)] - [N_i lnN_i - N_i + g_i lng_i - g_i]\}$$

$$= \Sigma\{(N_i + g_i)\ ln(g_i + N_i) - \cancel{N_i} - \cancel{g_i} - N_i lnN_i + \cancel{N_i} - g_i lng_i + \cancel{g_i}\}$$

$$= \Sigma\{N_i ln(g_i + N_i) + g_i ln(g_i + N_i) - N_i ln\ N_i - g_i lng_i\}$$

$$ln\ W = \Sigma\left\{N_i ln\left(\frac{g_i+N_i}{N_i}\right) + g_i ln\left(\frac{g_i+N_i}{g_i}\right)\right\}$$

$$ln\ W = \Sigma\left\{N_i ln\ \left(1 + \frac{g_i}{N_i}\right) + g_i ln\left(1 + \frac{N_i}{g_i}\right)\right\} \qquad \text{.......................... (4)}$$

Derivation of Bose – Einstein Statistics

$$ln\ \mathbf{W} = \Sigma\{N_i ln\left(1+\frac{g_i}{N_i}\right) + g_i ln\left(1+\frac{N_i}{g_i}\right)\} \quad .... (4) \qquad [\text{where } \delta ln\ x_i = \frac{1}{x_i}\delta\ x_i]$$

- On differentiating, we get

$$\delta\ \mathbf{ln\ W} = \Sigma\{ln\left(1+\frac{g_i}{N_i}\right)\delta N_i + N_i \delta ln\left(1+\frac{g_i}{N_i}\right) + ln\left(1+\frac{N_i}{g_i}\right)\delta g_i + g_i \delta ln\left(1+\frac{N_i}{g_i}\right)\}$$

$$= \Sigma\{ln\left(1+\frac{g_i}{N_i}\right)\delta N_i + N_i \frac{N_i}{g_i+N_i}\delta\left(\frac{g_i+N_i}{N_i}\right) + ln\left(1+\frac{N_i}{g_i}\right)\delta g_i + g_i \frac{g_i}{g_i+N_i}\delta\left(\frac{g_i+Ni}{g_i}\right)\}$$

$$= \Sigma\{ln\left(1+\frac{g_i}{N_i}\right)\delta N_i + \cancel{N_i}\frac{\cancel{N_i}}{g_i+N_i}\left(-\frac{g_i}{\cancel{N_i^2}}\right)\delta N_i + \cancel{g_i}\frac{1}{\cancel{g_i}}\frac{g_i}{g_i+N_i}\delta N_i\} \quad [\text{where } \delta g_i = 0]$$

$$= \Sigma\{ln\left(1+\frac{g_i}{N_i}\right)\delta N_i \quad .... (5)$$

- Most Probable Distribution of particles
  - The one for which **W** is maximum (**W$_{max}$**)
  - Condition for maxima: **δW = δ ln W = 0** .... (6)
- Putting the condition of **eqn. 6**

$$\delta\ ln W = \Sigma\{ln\left(1+\frac{g_i}{N_i}\right)\delta N_i = 0 \quad .... (7)$$

Derivation of Bose – Einstein Statistics

- Distribution must satisfy the condition: N & E must remain constant,
  - δN & δE must be equal to zero

$$\delta N = \Sigma\, \delta N_i = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

$$\delta E = \Sigma\, \varepsilon_i\, \delta N_i = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

- Using **Lagrange's Method of Undetermined Multipliers**
  - multiplying **eqn. 8** by (α) and **eqn. 9** by (β)

$$\alpha\, \delta N = \Sigma\, \alpha \delta N_i = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$\beta\, \delta E = \Sigma\, \beta\, \varepsilon_i\, \delta N_i = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

$$\Sigma ln\left(1 + \frac{g_i}{N_i}\right) \delta N_i = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

  - adding **eqn.10 & 11** and subtracting **eqn. 7**, we get

$$\boxed{\Sigma\, [\alpha + \beta\, \varepsilon_i - ln\left(1 + \frac{g_i}{N_i}\right)]\, \delta N_i = 0} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

- $\delta N_0, \delta N_1, \delta N_2, \delta N_3, \ldots\ldots \delta N_i$ are independent of each other, so each term in summation must be zero $\delta N_i \neq 0$

$$\alpha + \beta\, \varepsilon_i - ln\left(1 + \frac{g_i}{N_i}\right) = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

$$ln\left(1 + \frac{g_i}{N_i}\right) = (\alpha + \beta\, \varepsilon_i) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

Derivation of Bose – Einstein Statistics

- Removing logarithm from **eqn. 14**

$$ln\left(1+\frac{g_i}{N_i}\right) = (\alpha + \beta\, \varepsilon_i) \quad \text{.............................} (14)$$

$$\left(1+\frac{g_i}{N_i}\right) = e^{(\alpha + \beta\varepsilon i)} \quad \text{.............................} (15)$$

$$\frac{g_i}{N_i} = e^{(\alpha + \beta\varepsilon i)} - 1$$ where $\beta = 1/kT$ **[k = Boltzmann constant]**

$$\frac{N_i}{g_i} = \frac{1}{e^{(\alpha + \beta\varepsilon i)} - 1} \quad \text{..............................} (16)$$

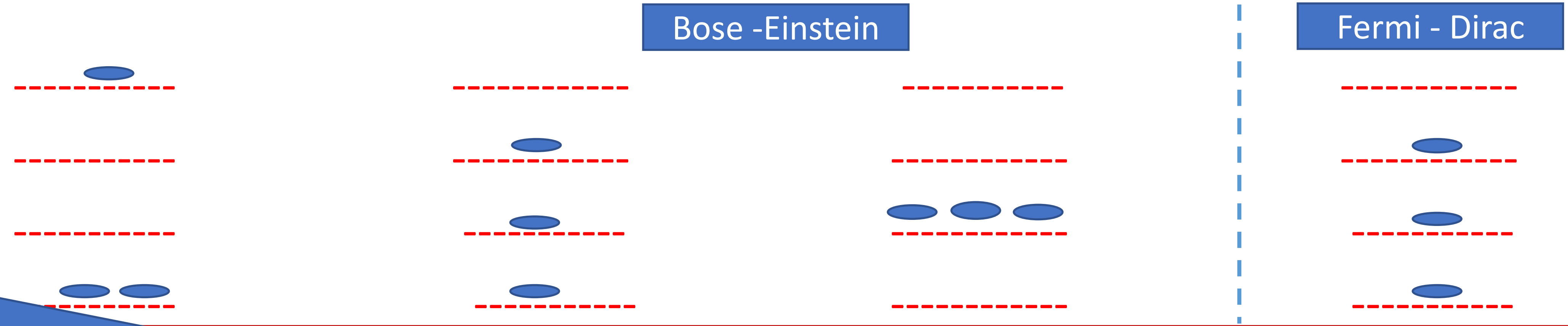
$$\boxed{N_i = \frac{g_i}{e^{(\alpha + \beta\varepsilon i)} - 1}} \quad \text{...............................} (17)$$

This equation gives Bose – Einstein Distribution

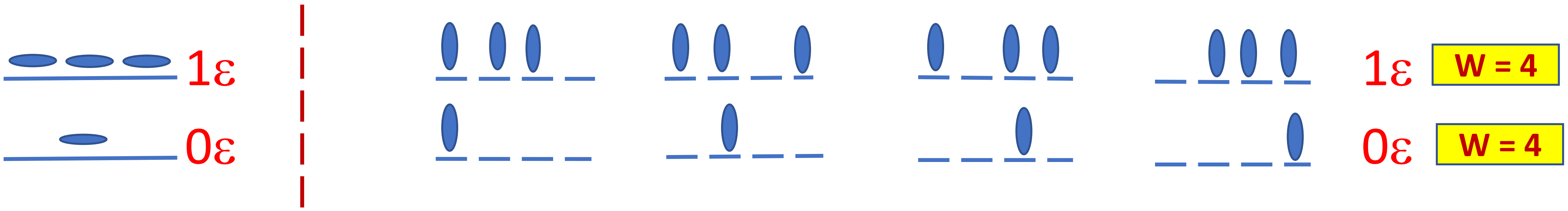
- **Application: Putting $e^{\alpha} = 1$ we can derive Planck’s Radiation Law**

FERMI - DIRAC STATISTICS - DERIVATION

- Suppose we have a bulk/ macroscopic system with the following conditions:
  - Consist of **N indistinguishable particles** with **total energy E** at **temperature T**
  - Total number of particles and energy must remain conserved
  - **Restriction** on **Occupancy** of energy states i.e., **Not more than one particle**
  - **Internal structure** taken into consideration
  - Particle have half integral spin e.g. Electron, proton, ………
  - The wave function is antisymmetric
  - Such particles are called **Fermions**



- Consider **4** particles distributed in **2** energy levels in such a way that there are **3** particles in 1ε and **1** particle in 0ε
- Now if each energy levels have **4** energy states, and with restriction that **only 1** particle occupy each energy state, then no. of microstates will be:



- Fermi-Dirac Statistics: **W = 4 x 4 = 16** (Pauli's Exclusion Principle)

- Bose – Einstein Statistics: **W = 20 x 4 = 80** (No restriction)

Derivation of Fermi – Dirac Distribution

- Most Probable Distribution **???**
  - The one with maximum no. of microstates - gives Maximum Thermodynamic Probability (**W_{max}**)
  - **$\delta W(N) = 0$ or $\delta \ln W(N) = 0$**
- Consider the distribution of **N** identical and indistinguishable particles among various energy levels - $\varepsilon_0, \varepsilon_1, \varepsilon_2$ .... having g_0, g_1, g_2, ... energy **states** with Total energy **E** at temperature **T, with not more than one particle in each energy state (Pauli's Exclusion Principle)**
- Total number of particles **N** and Total energy **E** remains **constant**

$$\Sigma N_i = N \quad \text{and} \quad \Sigma N_i \varepsilon_i = E$$

$$\Sigma\, \delta N_i = 0 \quad \text{and} \quad \Sigma\, \varepsilon_i\, \delta N_i = 0$$

- **N_0** particles are present in ε_0 energy level with g_0 energy states, **N_1** in ε_1 energy level with g_1 energy states, **N_2** in ε_2 energy level with g_2 energy states, ... **N_i** in ε_i energy level with g_i energy states with **only one particle per energy state**

Derivation of Fermi – Dirac Statistics

- Suppose there are N_i particles present in ε_i energy level
- The energy level ε_i is considered to be degenerate, in which there are g_i energy states where $g_i >>> N_i$
- N_i particles has to be arranged in ε_i energy level with g_i energy states
- **Restriction** on no. of particles i.e., only 1 particle occupy per energy state
- Permutations for g_i energy state will be $g_i!$
- Particles are **identical and indistinguishable**
- Permutations of N_i particles amongst themselves and $(g_i - N_i)$ vacant energy state amongst themselves has to be included
- Actual no. in which N_i particles may be allocated in ε_i energy level will be given by

$$\frac{g_i!}{N_i!\,(g_i - N_i)!}$$

Example

The Thermodynamic Probability will be given by:

$$W = \prod \frac{g_i!}{N_i!\,(g_i - N_i)!}$$

- $W = \dfrac{g_i!}{N_i!\,(g_i - N_i)!} = \dfrac{4!}{3!(4-3)!} = \dfrac{4\,!}{3!(1)!} = \dfrac{4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1}(1)} = 4$

- $W = \dfrac{g_i!}{N_i!\,(g_i - N_i)!} = \dfrac{4!}{1!(4-1)!} = \dfrac{4\,!}{1!(3)!} = \dfrac{4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{1(\cancel{3} \times \cancel{2} \times \cancel{1})} = 4$

- $W = \prod \dfrac{g_i!}{N_i!\,(g_i - N_i)!} = 4 \times 4 \quad = 16$

Derivation of Fermi – Dirac Statistics

- The Thermodynamic Probability will be given by :

$$\mathbf{W} = \prod \frac{g_i!}{N_i!\,(g_i - N_i)!} \qquad \text{.......................} \ (1)$$

- Taking logarithm, we get

$$ln\ \mathbf{W} = \Sigma\{\ln g_i! - [ln(N_i!) + ln\ (g_i - N_i)!]\} \qquad \text{.......................} \ (2)$$

- Applying **STIRLING'S Approximation:** $ln(N_i!) = N_i lnN_i - N_i$, we get

$$ln\ \mathbf{W} = \Sigma\{[g_i ln g_i - g_i] - [N_i lnN_i - N_i + (g_i - N_i)ln(g_i - N_i) - (g_i - N_i)]\}$$

$$= \Sigma\{g_i ln g_i - \cancel{g_i} - N_i lnN_i + \cancel{N_i} - (g_i - N_i)ln(g_i - N_i) + \cancel{g_i} - \cancel{N_i}\}$$

$$= \Sigma\{g_i ln g_i - N_i lnN_i - g_i ln(g_i - N_i) + N_i ln(g_i - N_i)\}$$

$$ln\ \mathbf{W} = \Sigma\{g_i ln\left(\frac{g_i}{g_i - N_i}\right) + N_i ln\left(\frac{g_i - N_i}{N_i}\right)\}$$

$$ln\ \mathbf{W} = \Sigma\{N_i ln\left(\frac{g_i}{N_i} - 1\right) - g_i ln\left(\frac{g_i - N_i}{g_i}\right)\}$$

$$ln\ \mathbf{W} = \Sigma\{N_i ln\left(\frac{g_i}{N_i} - 1\right) - g_i ln\left(1 - \frac{N_i}{g_i}\right)\} \qquad \text{.......................} \ (3)$$

Derivation of Fermi – Dirac Statistics

$$ln\, \mathbf{W} = \Sigma\{N_i ln\left(\frac{g_i}{N_i}-1\right) - g_i ln\left(1-\frac{N_i}{g_i}\right)\}\text{...............} (3) \quad [\text{where } \delta ln\, x_i = \frac{1}{x_i}\delta\, x_i]$$

- On differentiating, we get

$$\delta\, \mathbf{ln\, W} = \Sigma\{ln\left(\frac{g_i}{N_i}-1\right)\delta N_i + N_i \delta ln\left(\frac{g_i}{N_i}-1\right) - ln\left(1-\frac{N_i}{g_i}\right)\delta g_i - g_i \delta ln\left(1-\frac{N_i}{g_i}\right)\}$$

$$= \Sigma\{ln\left(\frac{g_i}{N_i}-1\right)\delta N_i + N_i \frac{N_i}{g_i-N_i}\delta\left(\frac{g_i-Ni}{N_i}\right) - g_i \frac{g_i}{g_i-N_i}\delta\left(\frac{g_i-Ni}{g_i}\right)\} \quad [\text{where } \delta g_i = 0]$$

$$= \Sigma\{ln\left(\frac{g_i}{N_i}-1\right)\delta N_i + \cancel{N_i} \frac{\cancel{N_i}}{g_i-N_i}\left(-\frac{g_i}{\cancel{N_i^2}}\right)\delta N_i - g_i \frac{\cancel{g_i}}{g_i-N_i}\left(-\frac{\cancel{g_i}}{\cancel{g_i^2}}\right)\delta N_i\}$$

$$= \Sigma\{ln\left(\frac{g_i}{N_i}-1\right)\delta N_i - \cancel{\frac{g_i}{g_i-N_i}}\delta N_i + \cancel{\frac{g_i}{g_i-N_i}}\delta N_i\}$$

$$= \Sigma\, ln\left(\frac{g_i}{N_i}-1\right)\delta N_i \qquad \text{..........................} (4)$$

- Most Probable Distribution of particles
  - The one for which W is maximum (W_{max})
  - Condition for maxima: $\delta W = \delta \ln W = 0$ .... (5)
- Putting the condition of **eqn. 5**

$$\delta\, ln W = \Sigma\, ln\left(\frac{g_i}{N_i}-1\right)\delta N_i = 0 \qquad \text{...........................} (6)$$

Derivation of Fermi – Dirac Statistics

- Distribution must satisfy the condition: N & E must remain constant,
  - δN & δE must be equal to zero

$$\delta N = \Sigma\, \delta N_i = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$\delta E = \Sigma\, \varepsilon_i\, \delta N_i = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

- Using **Lagrange's Method of Undetermined Multipliers**
  - multiplying **eqn. 7** by (α) and **eqn. 8** by (β)

$$\alpha\, \delta N = \Sigma\, \alpha \delta N_i = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

$$\beta\, \delta E = \Sigma\, \beta\, \varepsilon_i\, \delta N_i = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$\Sigma\, ln\left(\frac{g_i}{N_i} - 1\right) \delta N_i = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

  - adding **eqn. 9 & 10** and subtracting **eqn. 6**, we get

$$\boxed{\Sigma\, [\alpha + \beta\, \varepsilon_i - ln\left(\frac{g_i}{N_i} - 1\right)]\, \delta N_i = 0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

- $\delta N_0, \delta N_1, \delta N_2, \delta N_3, \ldots\ldots\, \delta N_i$ are independent of each other, so each term in summation must be zero $\delta N_i \neq 0$

$$\alpha + \beta\, \varepsilon_i - ln\left(\frac{g_i}{N_i} - 1\right) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

$$ln\left(\frac{g_i}{N_i} - 1\right) = (\alpha + \beta\, \varepsilon_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

- Removing logarithm from **eqn. 13**

$$\ln\left(\frac{g_i}{N_i} - 1\right) = (\alpha + \beta\,\varepsilon_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$\left(\frac{g_i}{N_i} - 1\right) = e^{(\alpha + \beta\varepsilon i)} \quad . \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

$$\frac{g_i}{N_i} = e^{(\alpha + \beta\varepsilon i)} + 1$$ where **$\beta = 1/kT$ [k = Boltzmann constant]**

$$\frac{N_i}{g_i} = \frac{1}{e^{(\alpha + \beta\varepsilon i)} + 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

$$\boxed{N_i = \frac{g_i}{e^{(\alpha + \beta\varepsilon i)} + 1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

This equation gives Fermi – Dirac Distribution

- **Application: Derive expression for heat capacity C_v of metals at low temperature**

COMPARISON BETWEEN THE THREE STATISTICS

Maxwell - Boltzmann	Bose - Einstein	Fermi - Dirac
•Classical Statistics	•Quantum Statistics	•Quantum Statistics
•Internal Structure ignored	•Internal Structure taken into account	•Internal Structure taken into account
•Identical & distinguishable particles	•Identical & indistinguishable particles	•Identical & indistinguishable particles
•No restriction on occupancy of energy states	•No restriction on occupancy of energy states	•Not more than one particle in each state –Pauli's Exclusion Principle
•Phase space not known	•Phase space is known ~ h^3	•Phase space is known ~ h^3
•Spin can have any value	•Zero or Integral Spin	• Half Integral Spin
•Wavefunction not involved	•Wavefunction Symmetric	•Wavefunction Antisymmetric
•At absolute zero, energy taken as zero	•At absolute zero, energy is taken to be zero	•At absolute zero, energy is not zero.
•Boltzmannons or Maxwellons	•Bosons	•Fermions
•Eg. Ideal gas molecules	•Eg. Photons, mesons, ^4He, ^2H, ….	•Eg. Electron, proton,…..
	•At high temperature approaches Maxwell-Boltzmann distribution	•At high temperature approaches Maxwell-Boltzmann distribution

COMPARISON BETWEEN THE THREE STATISTICS

Maxwell - Boltzmann

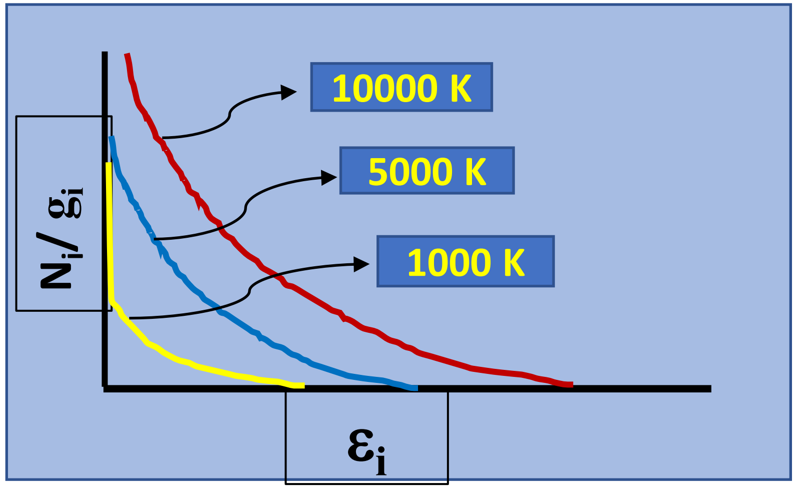
- No. of ways of achieving:

$$W = N!\prod \frac{g_i^{\,N_i}}{N_i!}$$

- Max. Probability distribution

$$\propto \frac{1}{e^{(\alpha + \beta\varepsilon_i)}}$$

- Distribution $N_i = \frac{g_i}{e^{(\alpha + \beta\varepsilon_i)}}$



Bose - Einstein

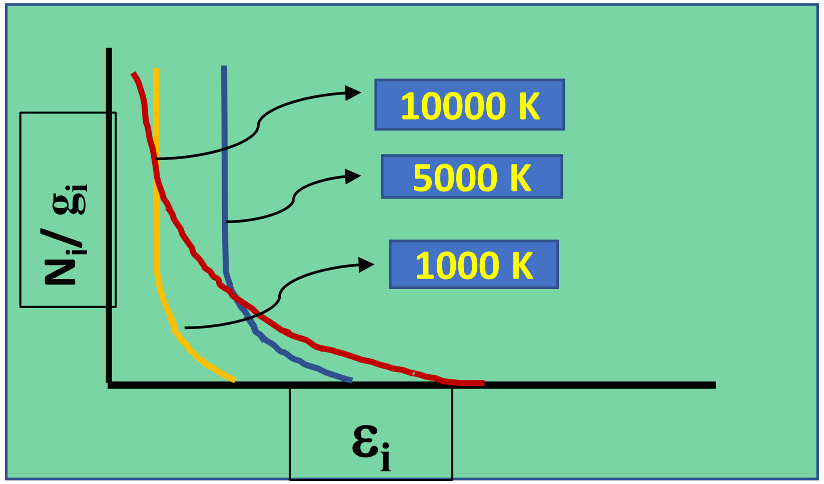
- No. of ways of achieving:

$$W = \prod \frac{(N_i + g_i - 1)!}{N_i!(g_i - 1)!}$$

- Max. Probability distribution

$$\propto \frac{1}{e^{(\alpha + \beta\varepsilon_i)} - 1}$$

- Distribution $N_i = \frac{g_i}{e^{(\alpha + \beta\varepsilon_i)} - 1}$



Fermi - Dirac

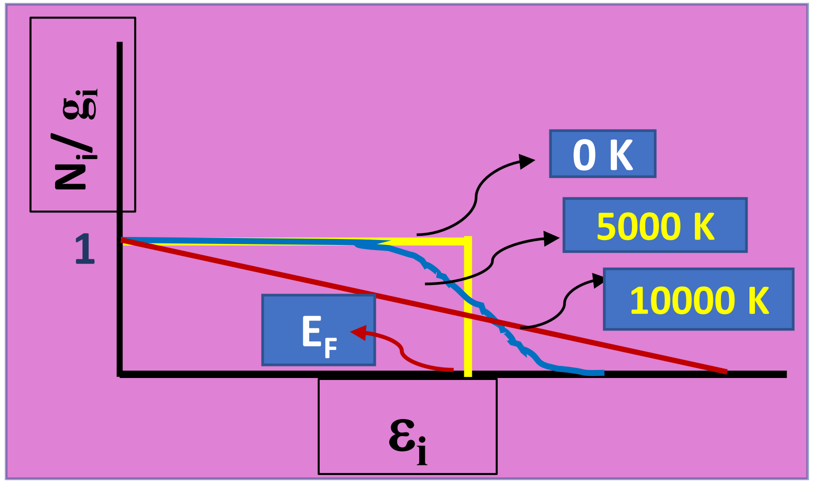
- No. of ways of achieving:

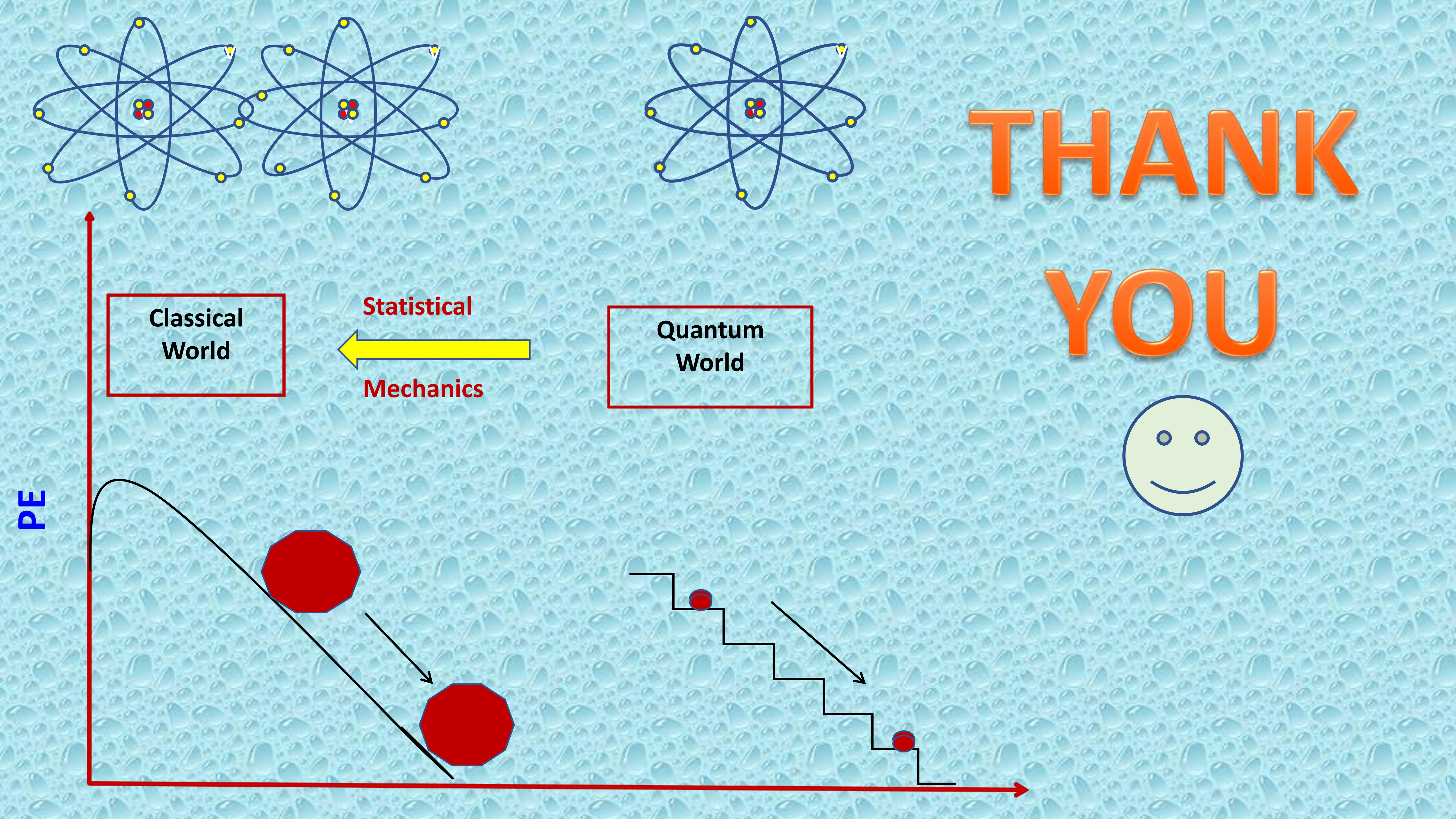
$$W = \prod \frac{g_i!}{N_i!\,(g_i - N_i)!}$$

- Max. Probability distribution

$$\propto \frac{1}{e^{(\alpha + \beta\varepsilon_i)} + 1}$$

- Distribution $N_i = \frac{g_i}{e^{(\alpha + \beta\varepsilon_i)} + 1}$

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THANK YOU