

## ANGULAR MOMENTUM OPERATOR

Angular momentum - Important quantity for rotating system

$$\vec{L} = \vec{r} \times \vec{p}$$

where  $r$  = radius vector

$p$  = linear momentum vector

$$\begin{vmatrix} i & j & k \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

In terms component:

$$\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z$$

$$\vec{p} = \vec{i}p_x + \vec{j}p_y + \vec{k}p_z$$

where  $\vec{i}, \vec{j}$  &  $\vec{k}$  are unit vectors along  $x, y, z$  axis

Thus  $\vec{L} = \vec{i}(\underbrace{y p_z - z p_y}_L)_x + \vec{j}(\underbrace{z p_x - x p_z}_L)_y + \vec{k}(\underbrace{x p_y - y p_x}_L)_z$

The classical expression of component  $L_z$  :

$$L_n = \gamma p_y - \beta p_x$$

The quantum mechanical operator will be

$$\hat{L}_n = \frac{\hbar}{2\pi i} \left[ \gamma \frac{\partial}{\partial y} - \beta \frac{\partial}{\partial x} \right]$$

$$\text{where } \hat{p} = \frac{\hbar}{2\pi i} \frac{\partial}{\partial q}$$

Similarly, we can get

$$\hat{L}_y = \frac{\hbar}{2\pi i} \left[ z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right]$$

$$\hat{L}_z = \frac{\hbar}{2\pi i} \left[ x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right]$$

$$q = x, y, z$$

The total angular momentum

$$\mathbf{L} = i\mathbf{L}_x + j\mathbf{L}_y + k\mathbf{L}_z$$

The scalar product is very useful quantity

$$\mathbf{L} \cdot \mathbf{L} = L^2 = L_x^2 + L_y^2 + L_z^2$$

Since for spherically symmetrical system, angular momentum is conserved, it is usually represented in spherical polar coordinates substituting the transformation results we get:

$$\hat{L}_x = \frac{\hbar}{2\pi i} \left[ -\sin\phi \frac{\partial}{\partial\theta} - \cot\theta \cos\phi \frac{\partial}{\partial\phi} \right]$$

$$\hat{L}_y = \frac{\hbar}{2\pi i} \left[ \cos\phi \frac{\partial}{\partial\theta} - \cot\theta \sin\phi \frac{\partial}{\partial\phi} \right]$$

$$\hat{L}_z = \frac{\hbar}{2\pi i} \frac{\partial}{\partial\phi} \quad \text{and} \quad \hat{L}^2 = -\frac{\hbar^2}{4\pi^2} \left[ \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left( \sin\theta \frac{\partial}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right]$$