PROBABILITY

Basic Concepts, Types and Applications

Probability

- Few centuries ago, gambling and gaming were considered to be fashionable and became widely popular among many men
- As the games became more complicated, players were interested in knowing the chances of winning or losing a game from a given situation.
- In 1654, Chevalier de Mere, a French nobleman with a taste of gambling, wrote a letter to one of the prominent mathematician of the time, Blaise Pascal, seeking his advice about how much dividend he would get for a gambling game played by paying money.
- Pascal worked this problem mathematically and share this problem to his good friend and mathematician Pierre de Fermat
- Their subsequent correspondences on the issue represented the birth of Probability Theory as a new branch of mathematics.

What is Probability?

- The way of expressing the knowledge of belief that an event will occur on chance
- It is originated from a Latin word means approval
- It is a branch of mathematics that studies pattern of chance
- It is based on observations that describes what happens after many, many trials

Terms used in Probability

1. Experiment: Is a situation involving chance and leading to result

2. Random Experiment

- A random experiment is an experiment in which

(i) The set of all possible outcomes are known

(ii) Exact outcome is not known

Example : 1. Tossing a coin. 2. Rolling a die. 3. Selecting a card from a pack of 52 cards

3. Sample space

The set of all possible outcomes in a random experiment is called a sample space.

- It is generally denoted by S

Example : When we roll a die, the possible outcomes are the face numbers 1,2,3,4,5,6 of the die. Therefore the sample space is S = {1,2,3,4,5,6}

4. Outcome: a possible result of a random experiment

5. Equally likely outcomes: All outcomes with equal probability

6. Tree diagram

- Tree diagram allow us to see visually all possible outcomes of an random experiment. Each branch in a tree diagram represent a possible outcome.

Illustration: (i) When we throw a die, then from the tree diagram the sample space can be written as S = {1,2,3,4,5,6}

(ii) When we toss two coins, then from the tree diagram the sample space can be written as S={HH,HT,TH,TT}

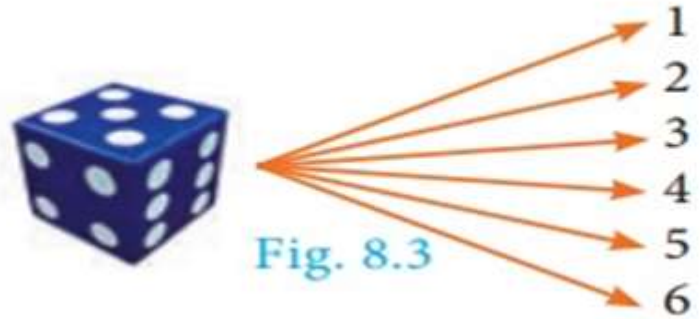


Fig. 8.3

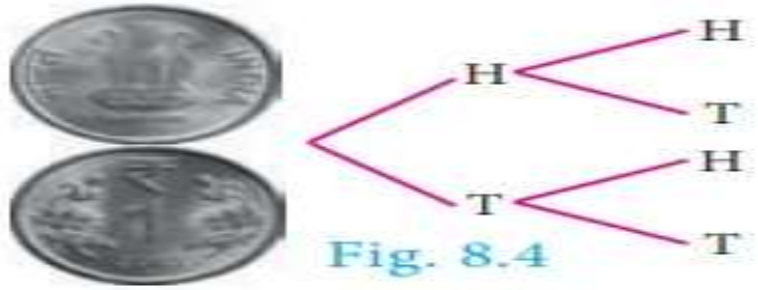


Fig. 8.4

7. Trial :

- Performing an experiment once is called a trial.

Example : When we toss a coin thrice, then each toss of a coin is a trial

8. Event:

- In a random experiment, each possible outcome is called an event. Thus, an event will be a subset of the sample space.

Example :Getting two heads when we toss two coins is an event

A. Simple events – An event has only one sample point. Occurrence or nonoccurrence of single event is simple event.

Ex. Drawing a particular card from a pack is simple event

B. Mixed or Compound or Joint events – an event has more than one sample points. Occurrence of two or more events simultaneously. If two person drawing one card each from a pack simultaneously, results of both kings are compound event

C. Equally Probable/ likely events – When two or more events are equally probable, the events are said to be Equally Probable. 50% chances of occurrence. Head and Tail are two Equally likely events.

D. Mutually Exclusive events – When occurrence of one event implies that the other cannot occur. Birth of male and female cannot occur simultaneously.

E. Sure event – Every set is subset of self event therefore, it Is said to be a sure event. It is also denoted by the symbol Ω.

F. Null or Impossible event – No chance of getting any event. Probability of getting 7 in tossing a dice. It is denoted by symbol ϕ

Probability

- It is the ratio of number of favorable cases to the total number of equally liked cases
- It is a vague concept which can not be defined mathematically
- It is a subjective value may change from person to person

$$P = \frac{Number\ of\ favorable\ cases}{Total\ number\ of\ equally\ likely\ cases}$$

- This study is to reduce the level of uncertainty in decision making
- So has special applications in business, administration and research

Probability

- It is the proportion of times an event occur in a set of trials

$$P = \frac{e}{t}$$

P- Probability

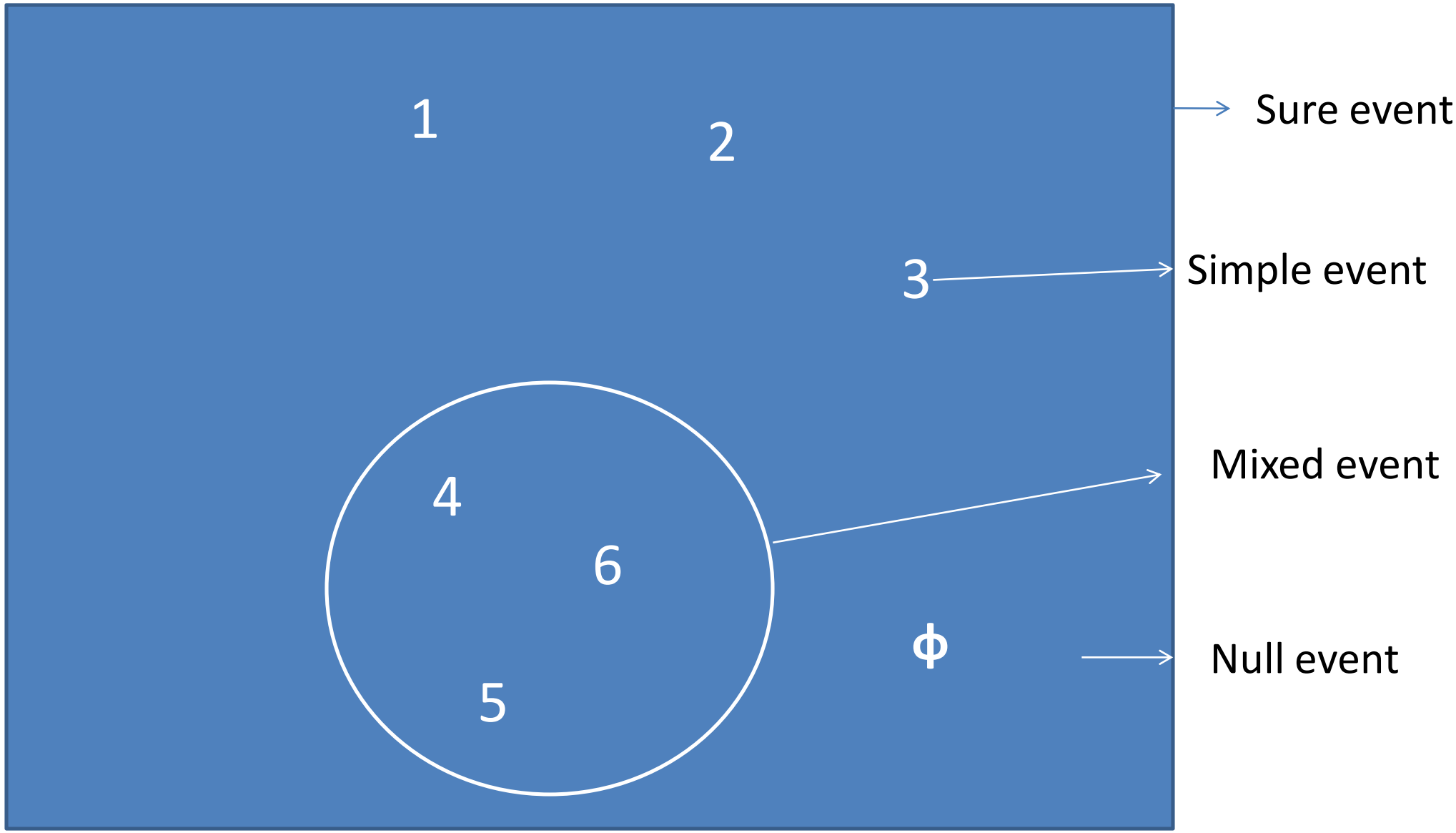
e- Number of times an event occurs /frequency

t- Total number of trials or items

The probability value is always a fraction falling between 1 and 0

- If P (A) > P (B) then A is more likely to occur,
- If P (A) = P (B) then both A and B are equally likely to occur

A Dice exhibiting different events



Kinds of Probability

1. **Classical or Mathematical Probability** – The probability is determined before the event has occurred
2. **Empirical or Statistical Probability** – The probability is determined after the event has occurred
3. **Theoretical probability** – The approach that bases the possibility of happening
4. **Subjective probability** – based on persons own reasoning and judgment

- If an event can happen in ‘a’ ways and fail to happen in ‘b’ ways and all these ‘a+b’ ways are equally likely, the probability of the happening of the event is measured by the ratio

$$\frac{a}{a+b} \quad \text{or} \quad \frac{b}{a+b}$$

If p is probability of event to occur and q is probability of event not occurring, so

$$p+q=1$$

$$q = 1-p$$

$$p = 1-q$$

Theorems/ Rules of Probability

1. **Additional Theorem** – It states that 2 events A and B are mutually exclusive, the probability of occurrence of either A or B is the sum of their individual probability

Ex. – A bag is containing 10 white, 10 black and 10 red balls. A ball is drawn at random. The probability of occurrence of white or red ball will be

Probability of white ball = 10/30 = 1/3

Probability of red ball = 10/30 = 1/3

Since the two events are mutually exclusive, occurrence of one excludes occurrence of other,

$$1/3 + 1/3 = 2/3$$

Answer = 2/3

Ex. – A bag containing 40 balls marked 1 to 40. The probability of drawing a number multiple of 8 and 10 will be calculated as

Probability of getting number multiple of 8

i.e. 8,16,24,32, 40 = 5/40

Ans. = 5/40

Probability of getting number multiple of 10

i.e. 10,20,30,40 = 4/40

Ans. = 4/40

2. Multiplication Theorem – If 2 events A and B are independent, the probability that they will both occur is equal to the product of their individual probability.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Ex. – A firm has advertised for a steno for its office with three qualities, English knowing, good looking and smart. Suppose the probability of English knowing is 1/50, good looking is 1/100 and smart is 1/1000. The probability of the firm getting such a steno will be

probability of English knowing is 1/50 = 0.02

probability of good looking is 1/100 = 0.01

probability of smart is 1/1000 = 0.001

Since the events are independent, the probability of simultaneously occurrence of the 3 qualities

= 1/50x 1/100x 1/1000 = 0.02x0.01x0.001 = 0.0000002

Ans. = 0.0000002

Probability from 2 independent events

Ex. – One bag contains 12 white and 20 black balls. A second bag contains 16 white and 24 black balls. One ball is drawn from each of the bag. The probability of both white outcomes will be –

The probability of white ball from first bag = 12/32

The probability of white ball from second bag = 16/40

The probability of both white outcomes from each of the bags

$$12/32 \times 16/40 = 6/40 = 3/20$$

Ans. 3/20

Applications

- Financial market
- Risk Assessment
- Environmental Regulation
- Scientific Analysis and Research
- Communication Network
- Signal Theory
- Information Technology