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Cherenkov radiation

Charged particle at rest or an uniform motion ⇒ does not radiate
⇒ But in moving with const. velocity ⇒ emits
⇒ It is unusual type of radiation.
first observed by Cherenkov, so its ~~Chee~~ Cherenkov radiation.
This type of radiation is
cooperative phenomenon
⇓
Involving large no. of atoms of the medium whose elec. are accelerated by the fields
⇓
emits radiation
this radiation is emitted by the medium under the action of passing particles. ⇒ emits radiation
Bremsstrahlung radiation ⇒ Particle has disappear large mass
Cherenkov radiation → visiable or unaffected

The wave vector & freq. of e.m. wave propogated in isotropic, non permeable medium ($K_m = 1$) are given by

$$k = \frac{\omega}{v} = \frac{n\omega}{c} \quad \text{where } n = \frac{c}{v} = \sqrt{K_e}$$
$$= \sqrt{\frac{\epsilon}{\epsilon_0}}$$

We know that the freq. of Fourier component of the field of a particle moving uniformly in X-direction in a medium is related to the x component of wave vector by $\omega = k_x v$

$\Rightarrow$ If it is freely propogated wave then

$$v = \frac{\omega}{k_x} > \frac{\omega}{k}$$

$$v > \frac{c}{n} \qquad \text{since } \frac{\omega}{k} = \frac{c}{n} \quad \text{——(1)}$$

n is function of ω

$\omega \to$ Cherenkov radiation of freq

$\Rightarrow$ So this freq occurs if the velocity of the particle exceeds the phase velocity of waves of that freq. in the medium

$$\therefore k_x = k\cos\theta = \left(\frac{n\omega}{c}\right)\cos\theta$$

$$k_x = \frac{\omega}{v}$$

$\theta \to$ angle betn the direction of motion of the particle and " " emission

so

$$\frac{\omega}{v} = \left(\frac{n\omega}{c}\right)\cos\theta \quad \text{——(2)}$$

$$\therefore \cos\theta = \frac{c}{nv} \quad \frac{c}{vn}$$

This relation indicates that definite value of angle θ corresponds to radiation of given freq. That is the radiation of each freq. is emitted forwards and its distributed over the surface of a cone of vertical angle 2θ

From Eq ②: $\theta \Rightarrow$ definite value (given freq.)

$\Rightarrow$ Radiation of freq. $\Rightarrow$ emits forwards

$\Downarrow$

distributed over the surface

$\Downarrow$

radiation + freq. are related in definite type

Isotropic – non permeable medium, ~~accordin~~ to Maxwell's equation takes the form

$D = \epsilon E, \quad B = \mu_0 H,$

$$\begin{cases} \operatorname{div} E = \dfrac{\rho}{\epsilon} \\ \operatorname{div} B = 0 \\ \operatorname{Curl} E = -\dfrac{\partial B}{\partial t} \\ \operatorname{Curl} H = \mu_0 J + \mu_0 \epsilon \dfrac{\partial E}{\partial t} \end{cases} \quad (X)$$

Maxwell's eqn

$$\begin{cases} \operatorname{div} D = \rho \\ \operatorname{div} B = 0 \\ \operatorname{Curl} E = -\dfrac{\partial B}{\partial t} \\ \operatorname{Curl} H = J + \dfrac{\partial D}{\partial t} \end{cases}$$

$n \to$ refractive index

For non permeable medium $n = \sqrt{K_e K_m}$

$$= \sqrt{\frac{\epsilon}{\epsilon_0} \cdot \frac{\mu_0}{\mu_0}} = \sqrt{\frac{\epsilon}{\epsilon_0}} = K_e$$

So charge $\rho = e\, \delta(r - vt)$ — 39 and current ... from

$J = ev\,\delta(r - vt)$ 3a

If ϕ = scalar potential

A = Vector

Then $B = \text{Curl } A$, $E = -\frac{\partial A}{\partial t} - \text{grad}\,\phi$ 3b

so the Maxwell's eqn (2) are satisfied identically

Now from the Lorentz condition (where A, ϕ are imposed)

$$\text{div } A + \mu_0 \epsilon \frac{\partial \phi}{\partial t} = 0$$

but $\mu_0 \epsilon_0 = \frac{1}{c^2}$, $\frac{\epsilon}{\epsilon_0} = k_e = n^2$

$$\text{div } A + \frac{n^2}{c^2} \frac{\partial \phi}{\partial t} = 0 \quad \text{— eq (4)}$$

Now using maxwell's eqn, two inhomogeneous eqns are

$$\left.\begin{aligned} \nabla^2 A - \mu_0 \epsilon \frac{\partial^2 A}{\partial t^2} &= \mu_0 J \\ \nabla^2 \phi - \mu_0 \epsilon \frac{\partial^2 \phi}{\partial t^2} &= \frac{\rho}{\epsilon} \end{aligned}\right\} \quad \text{— (5)}$$

equivalently above eqn can be written as

$$\left.\begin{aligned} \nabla^2 A - \frac{n^2}{c^2} \frac{\partial^2 A}{\partial t^2} &= -\mu_0 ev\,\delta(r - vt) \\ \nabla^2 \phi - \frac{n^2}{c^2} \frac{\partial^2 \phi}{\partial t^2} &= -\frac{e}{\epsilon}\,\delta(r - vt) \end{aligned}\right\} \quad (6)$$

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Now expanding A & ϕ as Fourier space Integral

$$\left.\begin{aligned} A &= \int_{-\infty}^{\infty} A_k\, e^{ik\cdot r}\, dk \\ \phi &= \int_{-\infty}^{\infty} \phi_k\, e^{ik\cdot r}\, dk \end{aligned}\right\} \quad (7)$$

Taking Laplacian of these values, we get

$$\left.\begin{aligned} \nabla^2 A &= -\int_{-\infty}^{\infty} A_k k^2\, e^{ik\cdot r}\, dk \\ \nabla^2 \phi &= -\int_{-\infty}^{\infty} \phi_k k^2\, e^{ik\cdot r} \end{aligned}\right\} - (8)$$

and the Fourier components of $\nabla^2 A$ & $\nabla^2 \phi$

$$\left.\begin{aligned} (\nabla^2 A)_k &= -k^2 A_k \\ (\nabla^2 \phi)_k &= -k^2 \phi_k \end{aligned}\right\} \quad - - - (9)$$

So the Fourier components of eqn (6)

$$k^2 A_k \; -A^2\phi_k - \frac{n^2}{c^2}\frac{\partial^2 A_k}{\partial t^2} = -\frac{1}{(2\pi)^3}\int \mu_0 e v\, \delta(r-vt)\, e^{-ik\cdot r}\, dv$$

$$= -\frac{1}{(2\pi)^3}\mu_0 e v\, e^{-ik\cdot vt} \quad --- (10a)$$

$$-k^2\phi_k - \frac{n^2}{c^2}\frac{\partial^2 \phi}{\partial t^2} = \frac{1}{(2\pi)^3}\int \mu_0 e v\, \delta(r-vt)\, e^{-ik\cdot r}\, dv$$

$$= -\frac{1}{(2\pi)^3}\mu_0 e v\, e^{ik\cdot vt} \quad (10b)$$

$$\cancel{-k^2\phi_k} - \frac{n^2}{c^2}\frac{\partial^2\phi_k}{\partial t^2}$$

Eq^n (10) may be written as

$$k^2A_k + \frac{n^2}{c^2}\frac{\partial^2 A_k}{\partial t^2} = \frac{\mu_0 e}{(2\pi)^3} v.\, e^{-ik.vt}$$

$$k^2\phi_k + \frac{n^2}{c^2}\frac{\partial^2\phi_k}{\partial t^2} = \frac{e}{(2\pi)^2 \epsilon} e^{-ik.vt}$$

So A & ϕ depends upon time factor $e^{-ik.vt}$

if $\omega = k.v = k_x v$

here we assume that the particle of charge e is moving along X-axis

so time factor $\rightarrow e^{-i\omega t}$

$$\frac{\partial^2}{\partial t^2} = -\omega^2$$

Putting $\epsilon = k_e \epsilon_0 = n^2 \epsilon_0$

Eq^n (11) can be written as

$$k^2A_k - \frac{n^2\omega^2}{c^2}A_k = \frac{\mu_0 e}{(2\pi)^3} v e^{-i\omega t}$$

$$k^2\phi_k - \frac{n^2\omega^2}{c^2}\phi_k = \frac{e}{(2\pi)^3 . n^2} e^{-i\omega t}$$

So the Fourier component A & ϕ are

$$A_k = \frac{\mu_0}{4\pi} \cdot \frac{e}{(2\pi)^2} \frac{V}{k^2 - \frac{\omega^2 n^2}{c^2}} e^{-i\omega t}$$

$$\phi_k = \frac{1}{4\pi\epsilon_0}\frac{e}{2\pi^2 n^2}\frac{1}{\left(k^2-\frac{\omega^2 n^2}{c^2}\right)} e^{-i\omega t}$$

From eqⁿ (5) the fourier component of electric field is given by

$\therefore E_k = i\omega A_k - ik\phi_k$

substituting value of A_k & ϕ_k

$$E_k = \frac{\mu_0}{2\pi}\frac{i\omega e}{(2\pi)^2}\cdot\frac{v}{k^2-\left(\frac{\omega^2 n^2}{c^2}\right)} e^{-i\omega t} - \frac{1}{4\pi\epsilon_0}\frac{ike}{2\pi^2 n^2}\cdot\frac{1}{\left[k^2-\frac{\omega^2 n^2}{c^2}\right]} e^{-i\omega t}$$

but $\mu_0\epsilon_0 = \frac{1}{c^2}$ or $\mu_0 = \frac{1}{\epsilon_0 c^2}$ we get

$$E_k = \frac{1}{4\pi\epsilon_0}\frac{ie\omega}{2\pi^2 n^2}\left[\frac{\frac{n^2}{c^2}v - \frac{k}{\omega}}{\left[k^2-\frac{\omega^2 n^2}{c^2}\right]}\right] e^{-i\omega t} \quad \text{--- (12)}$$

$\therefore$ Total field $E = \int_{-\infty}^{\infty} \frac{1}{4\pi} E_k e^{ik\cdot r} dk$

$$= \frac{1}{4\pi\epsilon_0}\frac{ie}{2\pi^2 n^2}\int_{-\infty}^{\infty}\frac{\left(\frac{n^2}{c^2}v - \frac{k}{\omega}\right)\omega}{\left(k^2-\frac{\omega^2 n^2}{c^2}\right)} e^{ik\cdot r} e^{-i\omega t} dk$$

$\therefore$ Force acting on the particle is

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$$F = eE$$

$$= \frac{1}{4\pi\epsilon_0} \frac{ie^2}{2\pi^2 n^2} \int_{-\infty}^{\infty} \frac{\left(\frac{n^2}{c^2} v - \frac{k}{\omega}\right)\omega}{\left(k^2 - \frac{n^2\omega^2}{c^2}\right)} e^{ik \cdot r} - e^{-i\omega t} \, dt$$

~~Now~~ let us find the value

The energy loss per unit length is the work done by the force

$$U = F$$

$\Downarrow$

It is stopping power of the substance w.r.t. the particle

~~dw~~ The energy loss in the freq. interval $d\omega$ is

$$dU = dF = -d\omega \cdot \frac{ie^2}{\pi} \Sigma\omega \left(\frac{1}{c^2} - \frac{1}{n^2 v^2}\right)$$

$$\int \frac{q\,dq}{q^2 - \omega^2\left(\frac{n^2}{c^2} - \frac{1}{v^2}\right)}$$

energy loss per unit length in the freq range $d\omega$ is

$$dU = dF = \frac{1}{4\pi\epsilon_0} \frac{e^2}{c^2} \left[1 - \frac{c^2}{n^2 v^2}\right] \omega \, d\omega$$

$\Downarrow$

This expression represents for the intensity of radiation in freq range $d\omega$

Cherenkov radiation is emitted only when $v > \frac{c}{n}$

which is in agreement with eqⁿ A

The total intensity of Chere. Radn. is obtained by integrating eqⁿ 2.