Unit- II

Electric and magnetic field due to uniformly moving charge

Stationary charge produces → E only

but uniformly moving " , E & B both are

but not radiate e.m. energy

So the e.m. energy can be radiated only if a charged particle is accelerated and it be computed by concept of retarded potential, the procedure to relate the cause & effect is known as retardation.

The electromagnetic fields of a uniformly Moving point charge

We know that el. & mag. fields of a point charge in motion are given by

$$E(r,t) = \frac{q}{4\pi\epsilon_0}\left[\frac{(\vec{R} - R\beta)(1-\beta^2)}{(R - \vec{R}\beta)^3} + \frac{(\vec{R} \times \vec{R} - R\beta) \times \dot{\beta}}{c(R - \vec{R}\beta)}\right]$$

$$B(r,t) = \frac{n \times E}{c} = \frac{\mu_0 c q}{4\pi}\left[\frac{(\beta \times n)(1-\beta^2)}{(R - \vec{R}\beta)^3} + \frac{n \times R \times (\vec{R} - }{cR(R - \vec{R}\beta)}\right]$$

These are the substitutions

$\mu_0\epsilon_0 = \frac{1}{c^2}$ $\quad \frac{d\vec{R}}{dt'} = -V.n \quad \frac{dR}{dt'} = -V$

$\beta = \frac{V}{c}$ $\quad \frac{d\beta'}{dt'} = \dot{\beta}$

$B = \frac{n \times E}{c}$

$V = c\beta$ Const. ⟹ uniformly moving pt charge

$\Rightarrow \dot{\beta} = 0$

for a uniformly moving point charge eq^n (i) takes the form

$$E(r,t) = \frac{q}{4\pi\epsilon_0}\left[\frac{(\vec{R} - R\beta)(1-\beta^2)}{(R - R\cdot\beta)^3}\right]_{retarded} \quad \ldots (3)$$

$\because$ Particle is moving with const. velocity,

$$\left.\begin{aligned} (\beta)_{retarded} &= \beta \\ \therefore\ [1-\beta^2]_{retarded} &= 1-\beta^2 \end{aligned}\right\} \quad \ldots (4)$$

So above eq^n (3) can be written as

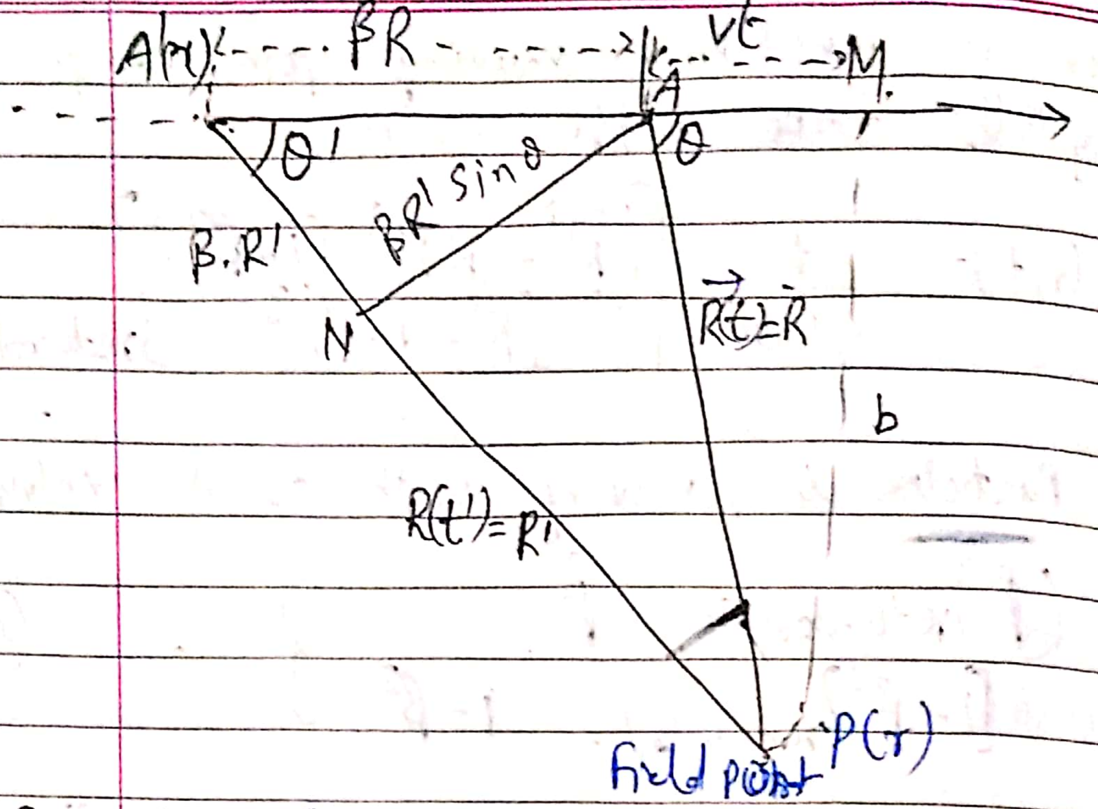
$$E(r,t) = \frac{q(1-\beta^2)}{4\pi\epsilon_0}\left[\frac{(\vec{R} - R\beta)}{\vec{R} - R\beta}\right]_{retarded} \quad \ldots (5)$$

The magnetic field may be computed by relation

$$B(r,t) = \left[\frac{n \times E(r,t)}{c}\right]_{retarded} \quad \ldots (6)$$

Eq^n (5) & (6) represents e.m. fields in terms of retarded position of point ch normal $R^{r} \rightarrow R_E$

Source Point



Point A → Present position of the charge
Point A' → retarded
(w. r. to point of observation P

$[R(t')] \Rightarrow$ retarded distance of field point P(r) from the source point A'(r')

$[R(t)] \Rightarrow$ Present distance of field point P from A

$\Rightarrow$ e.m. signal starting from source point A', reaches ~~the~~ the point P in in time ~~R~~ $\vec{R'}/c$,

$\Rightarrow$ charge moving with velocity $\vec{v}$ reaches the point A

Scanned with CamScan

$\theta \to$ angle between v & R
$\theta' \to$ " " v & R'

$AN \perp$ (perpendicular) from $A'P$
$PM \perp$ " P on the direction of velocity v

so from fig $\Delta AA'P$

$$[\vec{R'} - R\beta]_{\text{retarded}} = \vec{R'} - R'\beta = \vec{R} \quad \cdots (7)$$

$$b = R' \sin\theta' = R \sin\theta$$

$$\Rightarrow \beta R' \sin\theta' = \beta R \sin\theta$$

$$\Rightarrow \beta^2 R'^2 \sin^2\theta' = \beta^2 R^2 \sin^2\theta$$

$$\Rightarrow \beta^2 R'^2 (1 - \cos^2\theta') = \beta^2 R^2 (1 - \cos^2\theta)$$

$$\Rightarrow \beta^2 R'^2 - (\beta R' \cos\theta')^2 = \beta^2 R^2 - (\beta R \cos\theta)^2$$

$$\Rightarrow \beta^2 R^2 - (\vec{\beta} \cdot \vec{R'})^2 = \beta^2 R^2 - (\vec{\beta} \cdot \vec{R})^2 \quad \cdots (8)$$

Now from eqn (7) squaring

$$R'^2 - 2R'(\beta \cdot \vec{R'}) + R'^2 \beta^2 = R^2 \quad \cdots (9)$$

Eq (9) $-$ Eq (8)

$$R'^2 - 2R'(\vec{\beta} \cdot \vec{R'}) + (\vec{\beta} \cdot \vec{R'}) = R^2 - \beta^2 R^2 + (\vec{\beta} \cdot \vec{R})^2$$

$$(R' - \vec{R}' \cdot \beta)^2 = R^2 - \beta^2 R^2 + (\beta \cdot R$$

$$[\{R - \vec{R}\beta\}]_{retarded} = R'^2 - 2R'(\beta \cdot \vec{R}')^2 + (\beta \cdot \vec{R}')^2$$

$$= R^2 - \beta^2 R^2 + (\beta \cdot \vec{R})^2 \quad \ldots \quad (10)$$

$$= R^2 - \beta^2 R^2 + \beta^2 R^2 \cos^2\theta$$

$$= R^2 - \beta^2 R^2 (1 - \cos^2\theta)$$

$$= R^2 (1 - \beta^2 \sin^2\theta) \quad \ldots \quad (11)$$

now substituting values from eqn (2) and eqn (11) in eqn (5)

eqn (5)

$$E(r,t) = \frac{q(1-\beta^2)}{4\pi\epsilon_0} \left[\frac{(\vec{R} - R\beta)}{(R - \vec{R} \cdot \beta)} \right]_{retarded}$$

$$= \frac{q}{4\pi\epsilon_0} \cdot \frac{\vec{R}(1-\beta^2)}{R^3 (1 - \beta^2 \sin^2\theta)^{3/2}} \quad (12)$$

from eqn (2) $\dot{\beta} = 0$

$$B(r,t) = \left[\frac{n \times E}{c} \right]_{retarded}$$

$$= \frac{\mu_0 c q}{4\pi} \left[\frac{(\beta \times \vec{R})(1-\beta^2)}{(R - \vec{R} \cdot \beta)^3} \right]_{reta}$$

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from fig. $PM = R' \sin\theta'$

$= R \sin\theta$

$\Rightarrow$ It gives: $\beta R' \sin\theta' = \beta R \sin\theta$

$\vec{\beta} \times \vec{R'} = \beta \times \vec{R}$

$[\beta \times \vec{R}]_{retarded} = \vec{\beta'} \times \vec{R}$ ... (14)

from eqn (11) and (14), we get

$$B(r,t) = \frac{\mu_0 c q}{4\pi R^4} \frac{(\beta \times \vec{R})(1-\beta^2)^2}{(1-\beta^2 \sin^2\theta)^{3/2}}$$ ... (15)

$E(r,t)$ in eqn (12) and $B(r,t)$ in eqn (15) represents desired values of electromagnetic field vectors