Radiation damping - Abraham - Lorentz formula

When the charges moving in an external force field

$\Downarrow$

emit radiation

$\Downarrow$

carries off energy, momentum, ang. mome.

$\Downarrow$

there is reaction of radiation

From Larmor's formula (Power radiated) or the rate of energy loss due to radiation by a charged particle is

$$P = \frac{1}{4\pi\epsilon_0} \frac{2e^2 \dot{v}^2}{3c^3} \quad \text{—} \ (1)$$

If P. causes charge e, with magni accleration a, and time period T, then energy radiated

$$E_{rad} = \frac{1}{4\pi\epsilon_0} \left(\frac{2e^2 a^2 T}{3c^3} \right)$$

If this loss in energy is comparable to relevant energy E_0 then radiation reaction effective

when $E_{rad} \geq E_0$

but $E_0 = m (aT)^2$

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$\therefore E_{rad} - E_0$

$$= \frac{1}{4\pi\epsilon_0}\cdot\left[\frac{2e^2a^2T}{c^3}\right] - m(aT)^2 \quad — (2)$$

$$\therefore T = \frac{1}{4\pi\epsilon_0}\frac{2e^2}{3mc^3} \simeq \tau \quad — (3)$$

for $E_{rad} = E_0$

where $\tau \rightarrow$ characteristic time

therefore when

1 $T >> \tau$ radiative effect are no important, in that case $E_0 >> E_{rad}$

2 $T = \tau$ radiative effect are important so that when force is applied suddenly and for a short time the radiative effect will modify the motion of charged particle

So it may be concluded that the radiative reaction on the motion of charged particle is important if the external forces are such that the motion changes in time of order τ or over distance of the order of ~~$c\tau$~~ $c\tau$

If the motion of charged particle is periodic with freq ω and amplitude d, then energy of its motion

$$E_0 = m\omega_0^2 d^2$$

the acc. are $a \simeq \omega_0^2 d$

time interval $T \simeq 1/\omega_0$

subsituting the value of a & T in (2)

$$\therefore E_{rad} = E_0 = \frac{1}{4\pi\epsilon_0} \frac{2e^2 (\omega_0^4 d^2 \, 1/\omega_0)}{3c^3}$$

$$— m(\omega_0^2 d \cdot 1/\omega_0)^2$$

For radiation reaction to be effective criterion is $E_{rad} = E_0$

$$\therefore \frac{1}{4\pi\epsilon_0} \frac{2e^2 \omega_0^3 d^2}{3c^3} = m\omega_0^2 d^2$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \frac{2e^2 \omega_0}{3mc^3} = 1$$

from eqn (3) $\tau = \frac{1}{4\pi\epsilon_0} \cdot \frac{2e^2}{3mc^3}$

$$\Rightarrow \boxed{\omega_0 \tau = 1} \quad — (5)$$

ABRAHAM – Lorentz eqn of motion

Now we derive an eqn of motion of charged particles

we know from Newton's ~~eq.~~ equation of motion (from Newton's law)

$$F_{ext} = m\dot{v}$$

where → m → mass of charged particle

e → charge

F_{ext} → external force

since the particle is accelerated, it emits radiation, given by Larmor's form

$$P = \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{e^2}{c^3} (\dot{v})^2$$

to include its effect, we add a radiative reaction force F_{rad} in Newton's equation of motion, so that

$$F_{ext} + F_{rad} = m\dot{v} \quad — A$$

where F_{rad} must satisfy some of the following requirements-

1. it must be vanish if $\dot{v}=0$ since there is no radiation
2. $F_{rad} \propto e^2$ $\because$ radiative power $\propto e^2$
3. it should involve characteristic time τ.

for the motion of particle in any interval (t_1, t_2) energy should remain conserved though the radiative effects are present.

$\Rightarrow$ the workdone by F_{rad} on the particle in this interval $[t_1 < t < t_2]$ is equal to the negative of the energy radiated in that time i.e

$$\int_{t_1}^{t_2} F_{rad} \cdot v \, dt = -\int_{t_1}^{t_2} \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{e^2}{c^3} (\dot{v}\dot{v}) dt$$

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{c^3}$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{e^2}{c^3}\left[\int_{t_1}^{t_2} \ddot{v}\,v\,dt - (\dot{v}.v)\Big|_{t_1}^{t_2}\right]$$

but $\dot{v} \cdot v = 0$ if motion is periodic at $t = t_1$ & $t = t_2$

$$\int_{t_1}^{t_2}\left[F_{rad} - \frac{1}{4\pi\epsilon_0}\frac{e^2}{c^3}\cdot\ddot{v}\right]\cdot v\,dt = 0$$

∴ radiative reaction force is

$$F_{rad} = \frac{1}{4\pi\epsilon_0}\cdot\frac{2}{3}\frac{e^2}{c^3}\cdot\ddot{v}$$

$$= m\tau\ddot{v}$$

where $\tau = \frac{1}{4\pi\epsilon_0}\cdot\frac{2}{3}\frac{e^2}{mc^3}$

substituting in eqn A.

$$F_{ext} = m\dot{v} - m\tau\ddot{v}$$

$$\boxed{F_{ext} = m(\dot{v} - \tau\ddot{v})}$$

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Abraham-Lorentz equation of motion

if $F_{ext} = 0$, then its two solution

$$\dot{v}(t) = \begin{cases} 0 \\ a e^{t/\tau} \end{cases}$$

$(v \cdot v) \neq 0$. at t_1 & t_2

So the reactive term is very small and can cause only slow of small changes in the state of motion of charged particles