Modal equation:- The sol^n for β must be determined for boundary conditions. The boundary condition is required that tangential component of E and B inside and outside of an dielectric interface at $r=a$ must be same.

The eq^n

$$E_z = A J_\nu(ur)\, e^{j\nu\phi}\, e^{j(\omega t-\beta z)}$$

$$H_z = B J_\nu(ur)\, e^{j\nu\phi}\, e^{j(\omega t-\beta z)} \quad r<a$$

$$E_z = C K_\nu(\omega r)\, e^{j\nu\phi}\, e^{j(\omega t-\beta z)}$$

$$H_z = D K_\nu(\omega r)\, e^{j\nu\phi}\, e^{j(\omega t-\beta z)} \quad r>a$$

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Page :
Date : / /

Consider first the tangential component of E for z comp for eqn ① at the inner core cladding boundray $E_z = E_{z_1}$ and from eqn ③ at the out side of the boundary $E_z = E_{z_2}$ ie $E_{z_1} = E_{z_2}$

$$E_{z_1} - E_{z_2} = 0$$

$$A J_\nu(ua) - C K_\nu(\omega a) = 0 \quad -⑤$$

$$u^2 = K_1^2 - \beta^2$$

$$\omega^2 = \beta^2 - K_2^2$$

$$E_\phi = -\frac{j}{u^2}\left[\frac{\beta}{\cancel{r}}\frac{\partial E_z}{\partial \phi} - \mu\omega\frac{\partial H_z}{\partial r}\right] -⑥$$

From ①, ② and ⑥

$$E_{\phi_1} = \frac{-j}{u^2}\left[\frac{\beta}{r} A j\nu \, J_\nu(ua)\, e^{j\nu\phi} e^{j(\omega t - \beta z)} - \mu\omega B u J_\nu'(ua)\, e^{j\nu\phi} e^{j(\omega t - \beta z)}\right]$$

$-⑦$

Similarly E_{ϕ_2}

$$E_{\phi_1} - E_{\phi_2} = 0$$

$$= \frac{-j}{u^2}\left[A\frac{j\nu\beta}{a} J_\nu(ua) - B\omega\mu u J_\nu'(ua)\right]$$

$$\frac{-j}{\omega^2}\left[C\frac{j\nu\beta}{a} K_\nu(\omega a) - D\omega\mu\omega K_\nu'(\omega a)\right] = 0$$

$-⑧$

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Page :
Date : / /

Similarly $H_{z_1} - H_{z_2} = 0$

$$B J_\nu(ua) - D K_\nu(wa) = 0 \quad \text{--- (9)}$$

$$H_{\phi_1} - H_{\phi_2} = \frac{-j}{u^2}\left[\frac{Bj\nu\beta}{a} J_\nu(ua) + A\omega\varepsilon_1 u J_\nu'(ua)\right]$$

$$-\frac{j}{\omega^2}\left[\frac{Dj\nu\beta}{a} K_\nu(\omega a) + C\omega\varepsilon_2\, \omega K_\nu'(\omega a)\right] = 0 \quad \text{--- (10)}$$

Eq (7), (8), 9, (10) are set of four eqn with unknown Coeff. A, B, C D.

$$\begin{vmatrix} J_\nu(ua) & 0 & -K_\nu(wa) & 0 \\ \frac{\beta\nu}{u^2 a} J_\nu(ua) & \frac{j\omega\mu}{u} J_\nu'(ua) & \frac{\beta\nu}{a\omega^2} K_\nu(wa) & \frac{j\omega\mu}{\omega} K_\nu'(wa) \\ 0 & J_\nu(ua) & 0 & -K_\nu(wa) \\ \frac{-j\omega\varepsilon_1}{u} J_\nu'(ua) & \frac{\beta\nu}{au^2} J_\nu(ua) & \frac{-j\omega\varepsilon_2}{\omega} K_\nu'(\omega a) & \frac{\beta\nu}{a\omega^2} K_\nu(wa) \end{vmatrix} = 0$$

Soln of this determ. gives value of β

$$\left(\mathcal{J}_\nu + \mathcal{K}_\nu\right)\left(k_1^2 \mathcal{J}_\nu + k_2^2 \mathcal{K}_\nu\right) = \left(\frac{\beta\nu}{a}\right)^2 \left(\frac{1}{u^2} + \frac{1}{\omega^2}\right)^2$$

Here $\mathcal{J}_\nu = \dfrac{J_\nu'(ua)}{u J_\nu(ua)}$, $\mathcal{K}_\nu = \dfrac{K_\nu'(\omega a)}{\omega K_\nu(\omega a)}$

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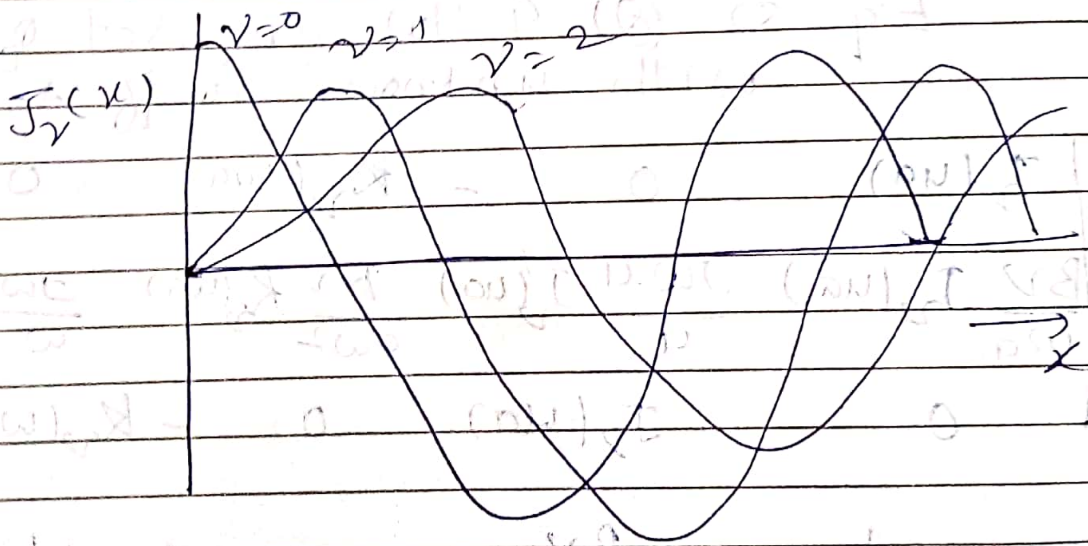
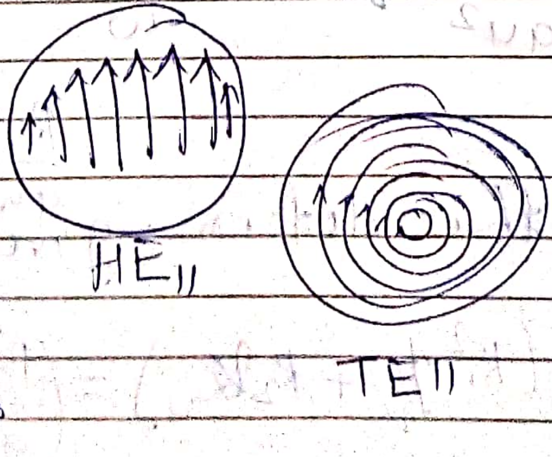
Solving eqn for β it will found that only discrete values restricted to the range given by $n_2 K < \beta < n_1 K$ will be allowed. Although eqn is a complicated eqn which is generally solved by numerical techniques. Its soln for any particular mode will provide all the ch. of that mode.

For $\nu = 0$ we get TE or TM mode

For $\nu \neq 0$ we get Hybrid mode HE or EH





TE_{01}

Page :
Date : / /

For the dielectric fiber waveguide all modes are hybrid modes except those for which $\nu = 0$ when $\nu = 0$

$$\left(\mathcal{J}_\nu + \mathcal{K}_\nu\right)\left(k_1^2 \mathcal{J}_\nu + k_2 \mathcal{K}_\nu\right) = 0 \quad (c)$$

$$\mathcal{J}_\nu + \mathcal{K}_\nu = 0$$

$$\frac{J_1(ua)}{u J_0(ua)} + \frac{K_1(wa)}{w K_0(wa)} = 0$$

This corresponds to TE_{om} modes ($E_z = 0$)

$$k_1^2 \mathcal{J}_\nu + k_2^2 \mathcal{K}_\nu = 0$$

$$\frac{k_1^2 J_1(ua)}{u J_0(ua)} + \frac{k_2^2 K_1(ua)}{w J_0(ua)} = 0$$

Corresponds to TM_{om} modes ($H_z = 0$)

When $\nu = 0$ the situation is different and numerical methods are used to solve (c)

ν	Mode	cutoff
0	TE_{om} or TM_{om}	$J_0(ua) = 0$
1	HE_{1m}, EH_{1m}	$J_1(ua) = 0$
≥ 2	$EH_{\nu m}$	$J_\nu(ua) = 0$